Diffraction and Interference

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Abstract: We measure the patterns produced by a CW laser near 650 nm passing through one and two slit apertures with a detector mounted on a linear translator. Data collected for a single slit of width $a = .04$ mm is used to measure the wavelength of light. Data collected for a double slit aperture of slit separation $d \sim .25$ mm and width $a \sim .04$ mm is fit with an algorithm that varies the parameters $a$ and $d$ in order to obtain a model that best describes the pattern. We report a wavelength of $716.15 \pm 10.06$ nm from the single slit data and parameters $a = .035$ mm and $d = .258$ mm from the double slit data.

I. Introduction

When monochromatic light waves pass through a slit or small opening, the light spreads out and interferes with itself to produce interesting patterns when viewed on a screen. This effect, known as diffraction, occurs because light passing through a certain part of the opening will interfere with light passing through other parts of the opening. Light interfering constructively or destructively produces bright spots and dark spots, respectively, along with intermediate intensities. Our objective for this experiment is to study the patterns produced by single-slit and double-slit openings, and compare the obtained data to a pattern predicted by theory using a non-linear fit program. We also aim to measure the wavelength of the light source by measuring the diffraction minima produced by the single slit.

Theory

The angular dependence of intensity for single and double slit patterns is well described by trigonometric functions. From interference conditions, we understand that single slit minima and double slit maxima are given by the following equations:

$$1 \text{ slit: } m\lambda = a \sin \theta, \quad m = 0, \pm 1, \pm 2 \ldots \quad (1)$$
$$2 \text{ slit: } m\lambda = d \sin \theta, \quad m = 0, \pm 1, \pm 2 \ldots \quad (2)$$

This is verified by plotting the theoretical functions describing single slit diffraction and double slit interference patterns, in terms of $a$ and $d$ (Figure 1).
For a given $\lambda$, the slit width, $a$, determines how widely spread the diffraction pattern is (Figure 1a). The slit separation determines how widely spread the interference pattern is (Figure 1b). In both cases, smaller widths produce more widely spread functions. This is often used as an analogy of the uncertainty relation in quantum mechanics.

\[ I = \sin^2\left(\frac{\pi a \theta}{\lambda}\right) \quad (3) \]

\[ I = \cos^2\left(\frac{\pi d \theta}{\lambda}\right) \quad (4) \]

\[ I = \sin^2\left(\frac{\pi a \theta}{\lambda}\right)\cos^2\left(\frac{\pi d \theta}{\lambda}\right) \quad (5) \]
II. Experimental Setup

We use a red laser near 650 nm as a coherent light source, and a stage with interchangeable slits is mounted in front of it. We use a single slit of width \( a = .04 \) mm and double slits of width \( a = .04 \) mm and separation \( d = .25 \) mm. In order to measure and quantify the pattern produced by the slits, we use a light detector 88 cm away on a linear track rather than a simple viewing screen. The stage is equipped with a detector to sensitively measure its linear position, such that we can sync optical intensity with x position. The set-up is shown in Figure 1.

![Figure 1: The set-up for the double slit experiment is shown. Slit width is represented by a, and slit separation by d. The filtering slit is attached to the detector in order to eliminate noise from neighboring points in the pattern.](image)

From the positions of the minima found for the single slit experiment, we aimed to calculate the wavelength of the laser using Eq. 1 and the labeled slit width of .04 mm. For the double slit experiment, we aimed to fit the obtained pattern to a theoretical model. Data from both detectors is fed to a computer, where Science Workshop plots them at a chosen sampling rate. Due to the intricate nature of the patterns predicted, we used a high sampling rate near 1000 Hz in order to clearly see all features. Data extracted from Science Workshop (in the form of linear position versus intensity) is put in terms of \( \theta \) using a measurement of \( L \) and plotted as scatter plots using Origin. The data points obtained for the double slit experiment are fitting using an algorithm in Origin that varies two parameters, \( a \) and \( d \), in order to best model the data.

III. Results

For the single slit set-up, we obtained 987 data points over a range of 11.6 cm of linear travel. This travel corresponds to a total angle of .131 radians: .069 radians to the right and .062 radians to the left. These values were chosen such that at least two peaks on either side of the central peak were included (Figure 3). All other data points had poor signal to noise ratio and were eliminated. Our single slit data produced six apparent minima which were used to calculate \( \lambda \) using Eq. 2 (Table 1). We report an average value for \( \lambda = 716.15 \pm 10.06 \) nm.
Figure 3: Data points for single slit experiment with approximate fit obtained by using known value \( a = .04 \) mm and calculated \( \lambda = 716.15 \).

\[
I = \text{sinc}^2\left(\frac{\pi (.04)\theta}{.000716}\right)
\]

<table>
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<tr>
<th>Linear Position (cm)</th>
<th>Angle from Center (rad.)</th>
<th>m</th>
<th>( \lambda ) (nm)</th>
<th>Error in ( \lambda )</th>
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<td>8.70</td>
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</tbody>
</table>

Table 1: Data points for measured minima are shown, along with calculation of \( \lambda \) and error in \( \lambda \) [1].

The data taken with the double slit aperture contained 2083 data points covering a linear range of 11.4 cm. This corresponds to an angle of .129 radians. Since the maximum angle of .066 radians is very small, the small angle approximation, \( \sin (\theta) \approx 0 \), is applied to the entire data set to speed up the fitting process. The fitting algorithm in Origin allowed us to set initial values for the two varying parameters, \( a \) and \( d \), and varied them such that \( \chi^2 \) was minimized. After fitting by hand to find adequate starting values for \( a \) and \( d \), we ran approximately 80 iterations of the algorithm to obtain a well-fitting function (Figure 4).
Figure 4: Double slit data (points) with best-fit model (red). Function plotted as best-fit also displayed.
IV. Discussion

Discontinuity arises in our results based on the inherent circularity of the two calculations of interest—one of wavelength and one of the two parameters $a$ and $d$. Because diffraction and interference produce patterns that depend on the ratios $a/\lambda$ and $d/\lambda$, respectively, there are an infinite number of combinations that could produce the patterns we observed. In calculating wavelength in the single slit experiment, we assume the slit created by Pasco is exactly .04 mm. Similarly, in varying the parameters $a$ and $d$ for the second experiment, we assume a light source of exactly 650 nm. The predicted ratio $a/\lambda = .04/\.00065 \sim 61.5$ gives an diffraction pattern too wide to fit either of our data sets, and calculations in both experiments produced ratios of 55.9 and 53.8. Since an infinite number of combinations of wavelengths and slit widths are capable of producing the pattern we observe, we are unable to know whether labeled values of the laser or slit are inaccurate. Another possible source of error is a discrepancy in the measurement of $L$. Although error in $L$ is taken into account in the calculation of $\lambda$, no portion of the apparatus was fastened to the table, allowing accidental changes in $L$ to become potentially significant.

In finding the “best-fit” model, we first matched diffraction minima and interference peaks to obtain rough estimates of $a$ and $d$. This was done to minimize the iterations needed to produce an adequate model. Since the algorithm adjusts parameters to minimize $\chi^2$, an initially poor model that needed to pass through variations that temporarily raised $\chi^2$ in order to ultimately minimize $\chi^2$ could cause the algorithm to collapse on an inadequate model by our standards. Furthermore, since noise levels prevented our data from reaching 0 at interference minima, the algorithm showed a tendency to overspread the fit model in order to reach these shallow dips. For these reasons, collaboration between our ability to recognize peak alignment and the computer’s ability to make fine adjustments was necessary to produce the most descriptive model.

References