Please staple this **coversheet** to your work. **Name:**____________________, ____________
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1.	2.	3.	4.	5.	6.	**Sum:**

Physics 311 **Homework Set 2** **Due: Sept. 3, 2009**

J. van Howe

1. A fluid rotates with angular velocity $\vec{\boldsymbol{\omega}} = \omega\hat{\mathbf{z}}$ about the z-axis.

(a) Find the velocity $\vec{\mathbf{v}}$ of any point in the fluid, and show that $\nabla\times\vec{\mathbf{v}} = 2\omega\hat{\mathbf{z}}$. Hint: the magnitude of the velocity is $\text{v} = \frac{ds}{dt}$, where s represents the arc length. Use this to write the velocity vector in cylindrical coordinates.

(b) Suppose that ω is a function of the (cylindrical) radius ρ: $\omega = a/\rho^2$, where *a* is a constant. Show that $\nabla\times\vec{\mathbf{v}} = 0$.

2. (a) Given $\vec{\boldsymbol{A}} = a\hat{\rho} + b\hat{\varphi} + c\hat{z}$ (cylindrical unit vectors), where a, b, c are constants, calculate $\nabla\times\vec{\boldsymbol{A}}$ and $\nabla\bullet\vec{\boldsymbol{A}}$. Does this make sense? Explain. Use mathematica or another program to plot this vector field.

(b) Given $\vec{\boldsymbol{A}} = a\hat{\boldsymbol{r}} + b\hat{\boldsymbol{\theta}} + c\hat{\varphi}$ (spherical unit vectors), where a, b, c are constants, calculate $\nabla\times\vec{\boldsymbol{A}}$ and $\nabla\bullet\vec{\boldsymbol{A}}$. Does this make sense? Explain. Use mathematica or another program to plot this vector field.

3. Griffiths 1.16

4. Find $\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}}$ in terms of the cylindrical unit vectors $\hat{\rho},\hat{\varphi},\hat{z}$. Do the same for $\hat{\boldsymbol{r}},\hat{\boldsymbol{\theta}},\hat{\varphi}$.

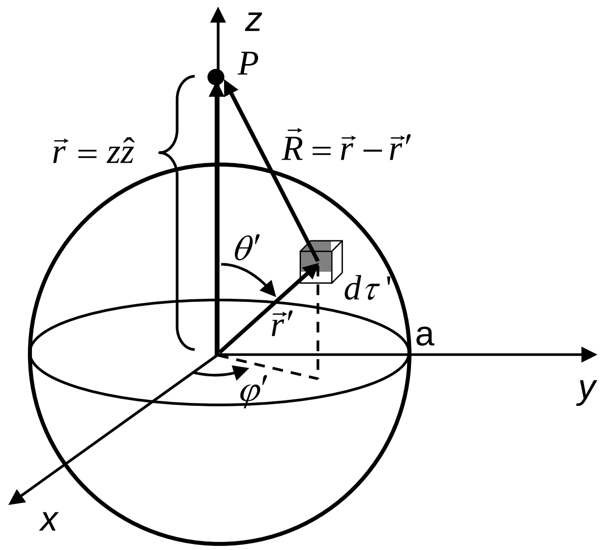
5. Derive the gradient formulas for cylindrical and spherical coordinates found at the front of your book.

6. Uniformly charged sphere, Problems 2.8, Ex. 2.2 and 2.11 in Griffiths

a) **Very nasty.** Using Coulombs law (Eq. 2.8), find the electric field at a point P located on the z-axis, due to a uniformly charged sphere ($\rho = const$) of radius **a (see the figure below)**. Do the case for inside the sphere and outside separately. This seemingly simple problem is quite nasty, so I will give you some strategy. The charge distribution has spherical symmetry, so we will integrate over spherical coordinates. However, one approach to simplify the problem is to write the position vector $\vec{\boldsymbol{R}}$ in terms of Cartesian unit vectors. Did you get that? Integrate over spherical coordinates, but use $\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}}$ as your basis. I'll let you in on another secret. Using the figure below, the x and y components of the field vanish. Make sure to justify this with the math as well as a common sense answer.

You may find this integral relation handy: $\int \frac{x-yu}{\left(x^2+y^2-2xyu\right)^{3/2}}du = \frac{xu-y}{x^2\left(x^2+y^2-2xyu\right)^{1/2}}$

After you obtain your expression, note that the total charge Q_{tot} on the sphere is related to the charge density ρ by $Q_{tot} = \int_{V_{sphere}} \rho(r', \theta', \varphi') d\tau'$. Use this expression to solve for ρ , and plug this into your result. Without loss of generality, you can substitute $z = r$. Does this expression make sense? Are you impressed or disappointed at all you did to arrive at this expression? Sketch a graph of the magnitude of the field vs. r .



b) **Nice n Easy.** Do the same problem using Gauss's law instead. Do your expressions match with part a)?