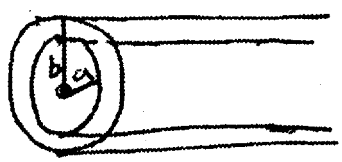
1. Griffiths 2.39

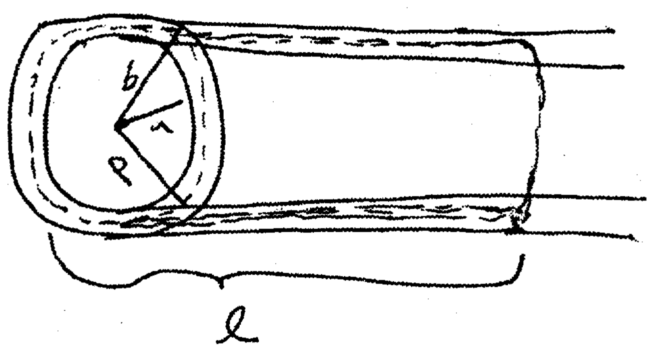


find capacitance per unit length

$\Rightarrow$ let's find capacitance for a given length l, and then we'll just divide through by l

$$C = \frac{Q}{V} \qquad V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$$

Find $\vec{E}$ using GAUSS'S LAW



--- cylinder is my GAUSSIAN Cylinder radius ρ and length l, surface area $= 2\pi\rho l$, and charge Q

$\vec{E}$ points in $\hat{\rho}$ direction $\quad \int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

$$\Rightarrow E \cdot 2\pi\rho l = \frac{Q}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{Q}{2\pi\rho l \varepsilon_0}\hat{\rho}}$$

1. cont.

$$V(b)-V(a) = -\int_a^b \frac{Q}{2\pi\rho\,l\varepsilon_0}\cdot d\rho = \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{a}{b}\right)$$

we put $+Q$ on inner cylinder so $V = V(a)-V(b)$

$$= \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

capacitance per unit length $= \frac{C}{l} = \boxed{\frac{2\pi\varepsilon_0}{\ln(b/a)}}$

2. Energy for sphere in #1

a) one way $\Longrightarrow$ $W = \frac{1}{2}\int \rho(r')V(r')\,d\tau'$

From problem 2.21 $\quad V(r') = \frac{Q_{Tot}}{4\pi\varepsilon_0 R}\left\{3 - \frac{r'^2}{R^2}\right\}$

last HW set

$$\rho(r') = \rho = \frac{Q_{Tot}}{4/3\,\pi R^3}$$

So $$W = \frac{Q_{Tot}}{8\pi\varepsilon_0 R}\cdot\frac{Q_{Tot}}{4/3\,\pi R^3}\cdot\frac{1}{2}\int_0^R\int_0^{2\pi}\int_0^{\pi}\left\{3 - \frac{r'^2}{R^2}\right\} r'^2 \sin\theta\, d\theta\, d\phi\, dr'$$

ϕ, θ integration

$$= \frac{Q_{Tot}}{8\pi\varepsilon_0 R}\,\frac{Q_{Tot}}{4/3\,\pi R^3}\,\frac{4\pi}{2}\int_0^R\left\{3 - \frac{r'^2}{R^2}\right\} r'^2\, dr'$$

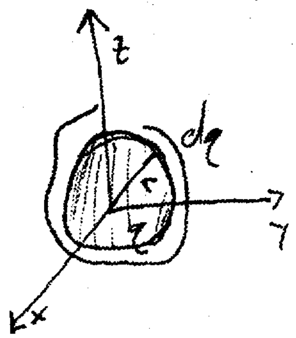
$$= \frac{Q_{Tot}^2}{4\pi\varepsilon_0}\,\frac{3}{4R^4}\left\{r'^3 - \frac{r'^5}{5R^2}\right\}\Bigg|_0^R$$

$$= \frac{Q_{Tot}^2}{4\pi\varepsilon_0}\,\frac{3}{4R^4}\left\{R^3 - \frac{R^3}{5}\right\}$$

$$\boxed{= \frac{1}{4\pi\varepsilon_0}\,\frac{3\,Q_{Tot}^2}{5\,R}}$$

2 b) Shell Method

$$dW = V\,dq \qquad \text{so that} \qquad W = \int_0^{Q_{TOT}} V\,dq$$



Shaded area is the sphere while it is being built. We give it a radius r and a charge of q. When we build up to $r = R$ we should have a total charge Q_{TOT}.

dq above is a little piece of infintesimal shell we add on until we get the whole sphere

During any given moment of building, the potential $V(r)$ looks like that of a pt. charge (we build from the origin out). As the sphere gets bigger it still looks like a pt. charge from outside:

$$V(r') = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}$$

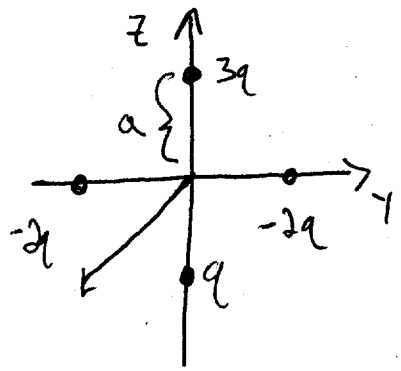
$$W = \int_0^{Q_{TOT}} \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\,dq$$

for any given sphere of charge q and radius r

$$q = \frac{4}{3}\pi r^3 \rho = \frac{Q_{TOT}\, r^3}{R^3}$$

since $\rho = \dfrac{Q_{TOT}}{4/3\,\pi R^3}$

3. Griffiths 3.27



all a distance a from origin

$$V_{dip}(r) = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\hat{r}}{r^2}$$

basically just need to find $\vec{p}$

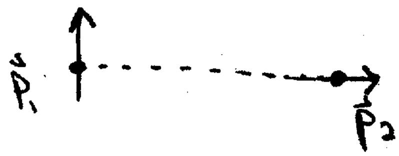
$$\vec{p} = \sum q_i \vec{r}_i{}' = 3qa\hat{z} + qa(-)\hat{z} - 2qa\hat{y} - (-)2qa\hat{y}$$

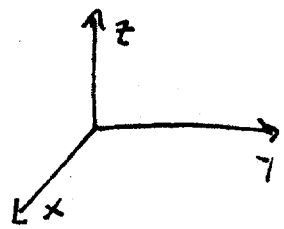
$$= 2qa\hat{z}$$

$$V_{dip}(r) = \frac{1}{4\pi\varepsilon_0} \frac{(2qa\hat{z})\cdot(\sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z})}{r^2}$$

$$\boxed{= \frac{1}{4\pi\varepsilon_0} \frac{2qa\cos\theta}{r^2}}$$

4. Griffiths 4.5





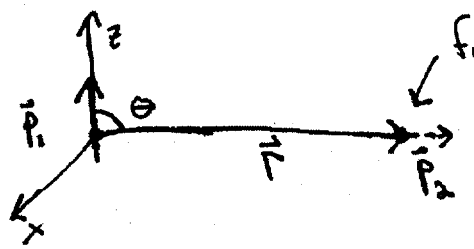
let's define direction

Torque on $\vec{p}_2$ due to field from $\vec{p}_1$ and visa-versa

$$\vec{N}_2 = \vec{p}_2 \times \vec{E}_{p_1}$$

$$E_{dipole} = \frac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

Griffiths 3.104



field point where I am interested

$$\vec{p}_1 = p_1\hat{z}$$

$\theta = 90°$ $\qquad \vec{p}_2 = p_2\hat{y}$

$$\vec{E}_{p_1}(r) = \frac{1}{4\pi\varepsilon_0 r^3}\left[3\underbrace{(p_1\hat{z}\cdot\hat{r})}_{p_1\cos\theta \to 0}\hat{r} - p_1\hat{z}\right]$$

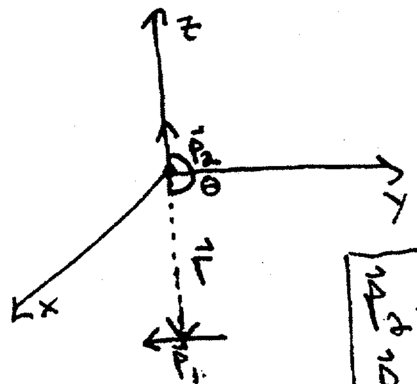
since $\theta = 90°$

$$\vec{E}_{p_1}(r) = \frac{-p_1\hat{z}}{4\pi\varepsilon_0 r^3}$$

$$\boxed{\vec{N}_2 = p_2\hat{y} \times \frac{-p_1}{4\pi\varepsilon_0 r^3}\hat{z} = \frac{-p_1 p_2}{4\pi\varepsilon_0 r^3}\hat{x}}$$

$$\vec{N}_1 = \vec{p}_1 \times \vec{E}_{p_2}$$

Though not necessary, when using the <u>general formula</u> Griffiths 3.104, I still like to move origin to the field-producing dipole so that it is in line with the z-axis as in the figure below. This is convention and <u>is necessary</u> if you use Griffiths 3.103 instead.



$$\boxed{\begin{aligned}\vec{p}_2 &= p_2\hat{z}\\ \vec{p}_1 &= -p_1\hat{y}\end{aligned}}$$

$$\vec{E}_{p2} = \frac{1}{4\pi\varepsilon_0 r^3}\left[3(p_2\hat{z}\cdot\hat{r})\hat{r} - p_2\hat{z}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left[3(p_2\underbrace{\cos\theta}_{-1})\hat{r} - p_2\hat{z}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left[3p_2\hat{z} - p_2\hat{z}\right]$$

$$= \frac{2p_2\hat{z}}{4\pi\varepsilon_0 r^3}$$

$\theta = 0$

$$\hat{r} = \underbrace{\sin\theta\cos\phi}_{0}\,\hat{x} + \underbrace{\sin\theta}_{0}\sin\phi\,\hat{y} + \underbrace{\cos\theta}_{-1}\,\hat{z}$$

since $\theta = 0$

$$\boxed{\hat{r} = -\hat{z}}$$

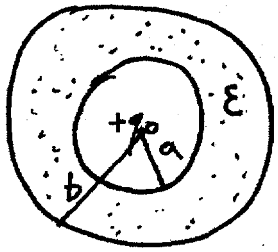
$$N_1 = \vec{p}_1 \times \vec{E}_{p2} = -p_1\hat{y} \times \frac{2p_2}{4\pi\varepsilon_0 r^3}\hat{z}$$

$$\boxed{N_1 = -\frac{2p_1p_2}{4\pi\varepsilon_0 r^3}}$$

twice as big as N_2! Weird.

Dipole depends on location of origin in respect to other dipoles

5.



a) $r < a$ No dielectric is present, No bound charge $\boxed{\vec{P} = 0}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E}$$

From Gauss's Law $\int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

$$\boxed{\vec{E} = \frac{q_0}{4\pi\varepsilon_0 r^2}\hat{r}}$$

so $\boxed{\vec{D} = \frac{q_0}{4\pi r^2}\hat{r}}$

$a < r < b$ Now we are inside dielectric $\vec{D}$ field easiest way to solve

$$\int \vec{D} \cdot d\vec{a} = Q_{f\,enc} \Rightarrow \boxed{\vec{D} = \frac{Q_{f\,enc}\hat{r}}{4\pi r^2} = \frac{q_0}{4\pi r^2}\hat{r}}$$

$\vec{D} = \varepsilon \vec{E}$ so $\boxed{\vec{E} = \frac{q_0}{4\pi\varepsilon r^2}\hat{r}}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \Rightarrow \vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \frac{q_0}{4\pi r^2} - \frac{\varepsilon_0 q_0}{4\pi\varepsilon r^2} = \underbrace{\boxed{\frac{q_0}{4\pi r^2}\left(1 - \frac{1}{\varepsilon_r}\right)\hat{r}}}_{\vec{P}}$$

5 cont

$r > b$

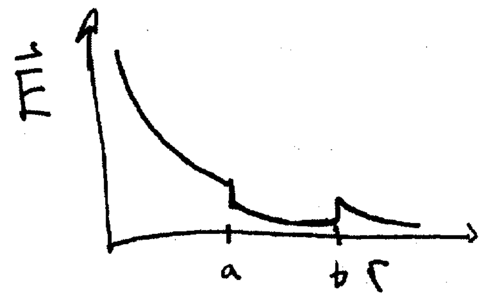
Gauss's Law for $\vec{D}$ field $\int \vec{D} \cdot d\vec{a} = Q_{f\,enc}$

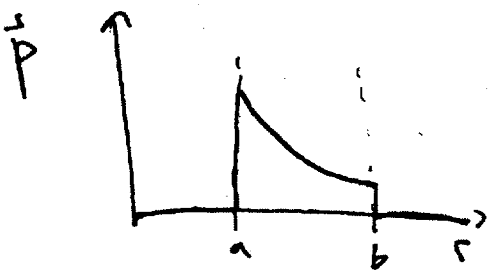
gives $\boxed{\vec{D} = \frac{q_0}{4\pi r^2}\hat{r}}$

Gauss's Law for $\vec{E}$ field gives $\int \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

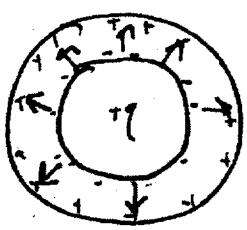
$$\boxed{\vec{E} = \frac{q_0}{4\pi\varepsilon_0 r^2}\hat{r}}$$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ $\quad \boxed{\vec{P} = \vec{D} - \varepsilon_0 \vec{E} = 0}$





b) $\sigma_b = \vec{P} \cdot \hat{n}$



similar to Griffiths 4.15

from a) we found
$\vec{P} = \frac{q_0}{4\pi r^2}\left(1 - \frac{1}{\varepsilon_r}\right)\hat{r}$

6. Griffiths 4.27

From Ex. 4.2 pg 168, find $\vec{E}$ field inside and out

$r < R$ $\quad \boxed{\vec{E} = -\frac{P}{3\varepsilon_0}\hat{z}}$

$r > R$ take gradient in spherical coords of V

$$\vec{E} = -\nabla V = -\nabla\left(\frac{P}{3\varepsilon_0}\frac{R^3}{r^2}\cos\theta\right)$$

from Ex 4.2

$$= -\frac{PR^3}{3\varepsilon_0}\left(\frac{\partial}{\partial r}\left(\frac{\cos\theta}{r^2}\right)\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\cos\theta}{r^2}\right)\hat{\theta}\right)$$

$$= \boxed{\frac{PR^3}{3\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$

Using 4.55

$$W = \frac{\varepsilon_0}{2} \int_{\text{all space}} E^2 \, d\tau$$

$$= \frac{\varepsilon_0}{2} \iiint_0^R E_{inside}^2 \, d\tau + \iiint_R^\infty (E_{out})^2 \, d\tau$$

$$= \frac{\varepsilon_0}{2} \frac{4}{3} \pi R^3 \left(\frac{P}{3\varepsilon_0}\right)^2 + \iiint_R^\infty (E_{out})^2 \, d\tau$$

This integral easy since E_{inside} has no r, θ or ϕ dependence $\rightarrow$ just a const

$$= \frac{2\pi P^2 R^3}{27 \varepsilon_0} + \underbrace{\int_0^{2\pi}\int_0^{\pi}\int_R^{\infty} \frac{\varepsilon_0}{2} \left(\frac{PR^3}{3\varepsilon_0}\right)^2 \frac{(4\cos^2\theta + \sin^2\theta)}{r^6} r^2 \sin\theta \, dr \, d\theta \, d\phi}$$

$$\frac{\varepsilon_0}{2} \frac{2\pi P^2 R^6}{9 \varepsilon_0^2} \int_0^{\pi} \int_R^{\infty} \left(\frac{4\cos^2\theta \sin\theta}{r^4} + \frac{\sin^3\theta}{r^4}\right) dr \, d\theta$$

$W_{R<r}$

$$\frac{\varepsilon_0}{2}\frac{2\pi}{9}\frac{P^2R^6}{\varepsilon_0^2}\int_0^{\pi}\int_R^{\infty}\frac{4((\cos\theta^2\sin\theta)+\sin\theta^3}{r^4}\,dr\,d\theta$$

$$=\frac{\pi P^2R^6}{9\varepsilon_0}\int_R^{\infty}\int_0^{\pi}\frac{4(1-\sin\theta^2)\sin\theta+\sin\theta^3}{r^4}\,d\theta\,dr$$

$$=\frac{\pi P^2R^6}{9\varepsilon_0}\int_R^{\infty}\int_0^{\pi}\frac{4\sin\theta-4\sin\theta^3+\sin\theta^3}{r^4}\,d\theta\,dr$$

$$=\frac{\pi P^2R^6}{9\varepsilon_0}\int_R^{\infty}\int_0^{\pi}\frac{4\sin\theta-3\sin\theta^3}{r^4}\,d\theta\,dr$$

we know

look up

$$=\frac{\pi P^2R^6}{9\varepsilon_0}\int_R^{\infty}\frac{8-4}{r^4}\,dr$$

$$=\frac{4\pi P^2R^6}{9\varepsilon_0}\int_R^{\infty}\frac{1}{r^4}\,dr$$

$$= \frac{4\pi \rho^2 R^6}{9\varepsilon_0} \left. \frac{1}{3r^3} \right|_\infty^R$$

$$= \frac{4\pi \rho^2 R^6}{27\varepsilon_0 R^3}$$

$$W_{tot} = \frac{2\pi \rho^2 R^3}{27\varepsilon_0} + \frac{4\pi \rho^2 R^3}{27\varepsilon_0}$$

$$= \frac{6\pi \rho^2 R^3}{27\varepsilon_0} = \boxed{\frac{2\pi \rho^2 R^3}{9\varepsilon_0}}$$

using 4.58

$$W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau = \frac{1}{2}\iiint_0^R D_{in}\cdot E_{in}\, d\tau + \frac{1}{2}\iiint_R^\infty D_{out} E_{out} d\tau$$

for $r > R$ $D = \varepsilon_0 E$ so same as above

$$W = \frac{1}{2}\iiint_0^R \vec{D}_{in}\cdot\vec{E}_{in}\, d\tau + \frac{4\pi \rho^2 R^3}{27\varepsilon_0}$$

$$\bar{D}_{in} = \varepsilon_0 \vec{E}_{in} + \bar{P}$$

$$D_{in} = -\frac{P}{3} + P = \frac{2}{3}P$$

so $\frac{1}{2}\iiint_0^R \vec{D}_{in} \cdot \vec{E}_{in} \, d\tau = \frac{1}{2}\iiint_0^R -\frac{2}{3}P \cdot \frac{P}{3\varepsilon_0} \, d\tau$

$$= \frac{1}{2}\left(\frac{4}{3}\pi R^3\right)\left(\frac{-2P^2}{9\varepsilon_0}\right)$$

$$= \frac{-4\pi R^3 P^2}{27\varepsilon_0}$$

$$W_{Tot} = -\frac{4\pi R^3 P^2}{27\varepsilon_0} + \frac{4\pi R^3 P^2}{27\varepsilon_0}$$

$= 0$ hmmm . . . .

- 1st method is correct electrostatic energy, but not total work needed to assemble since it leaves out mechanical energy needed to polarize the molecules

- Makes some sense since $\vec{D}$ depends on free charge and we have none However not the total work either since D is not path independent In this case, it all depends on how you put the sphere together