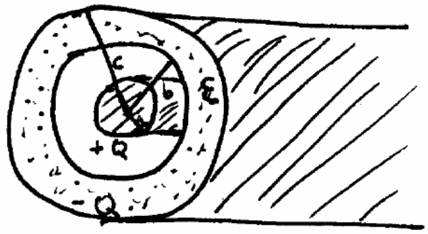
Griffiths 4.21



$$\frac{\text{Capacitance}}{\text{unit length}} = \frac{C}{\ell} = \frac{Q}{V\ell}$$

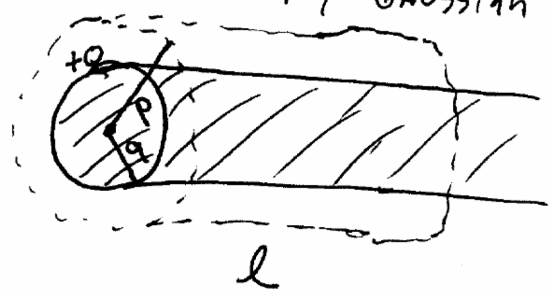
$$V = -\int_c^a \vec{E}\cdot d\vec{\ell}$$

So need to find $\vec{E}$ in regions between $a \to c$

put charge $+Q$ on inner conductor, by deff of capacitor, outer conductor will hold charge $-Q$

my GAUSSIAN surface

For $a < \rho < b$

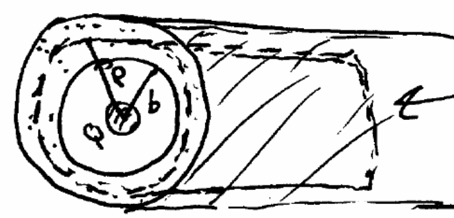


GAUSS's Law $\int \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$

$$\boxed{\vec{E} = \frac{Q}{2\pi\varepsilon_0\rho\ell}\hat{\rho}} = \vec{E}_1$$

for $b < \rho < c$ use GAUSS's Law for $\vec{D}$ field

$$\int \vec{D}\cdot da = Q_{fenc}$$



GAUSSIAN Cylinder

only free charge enclosed from inner conductor

$$\vec{D} = \frac{Q}{2\pi\ell\rho} \qquad \vec{D} = \varepsilon\vec{E} \quad \text{so} \quad \boxed{\vec{E} = \frac{Q}{2\pi\varepsilon\rho\ell}\hat{\rho}} = \vec{E}_2$$

So $V = -\int_c^a \vec{E} \cdot d\vec{l} = \int_a^c \vec{E} \cdot d\vec{l} = \int_a^b \vec{E}_1 \cdot dl + \int_b^c \vec{E}_2 \cdot dl$

$$= \int_a^b \frac{Q}{2\pi\varepsilon_0 \rho l} d\rho + \int_b^c \frac{Q}{2\pi\varepsilon \rho l} dl$$

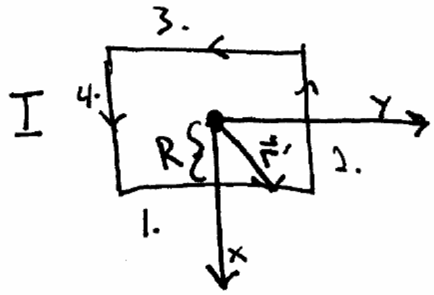
$$= \frac{Q}{2\pi l}\left[\frac{\ln(b/a)}{\varepsilon_0} + \frac{\ln(c/b)}{\varepsilon}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\left[\ln(b/a) + \frac{\varepsilon_0}{\varepsilon}\ln(c/b)\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 l}\left[\ln(b/a) + \frac{1}{\varepsilon_r}\ln(c/b)\right] \qquad \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\Rightarrow \frac{C}{l} = \frac{Q}{Vl} = \frac{2\pi\varepsilon_0}{\ln(b/a) + \frac{1}{\varepsilon_r}\ln(c/b)}$$

5.8 a) Griffiths



$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$

$\vec{\mathscr{r}} = -\vec{r}\,'$ since $\vec{r} = 0$
origin placed on field pt. of interest

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\,' \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

take one segment

1. $d\vec{l} = dy\,\hat{y}$

$\vec{\mathscr{r}} = -R\hat{x} - y\hat{y}$

$$d\vec{l} \times \vec{\mathscr{r}} = R\hat{z}$$

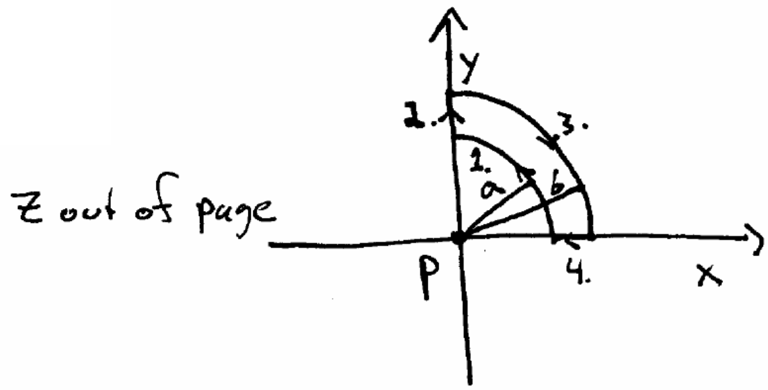
$$\mathscr{r}^3 = (R^2+y^2)^{3/2}$$

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi}\int_{-R}^{R} \frac{R\,\hat{z}\,dy}{(R^2+y^2)^{3/2}} = \frac{\mu_0 I}{4\pi}\frac{R}{R\sqrt{R^2+y^2}}\Big|_{-R}^{R}$$

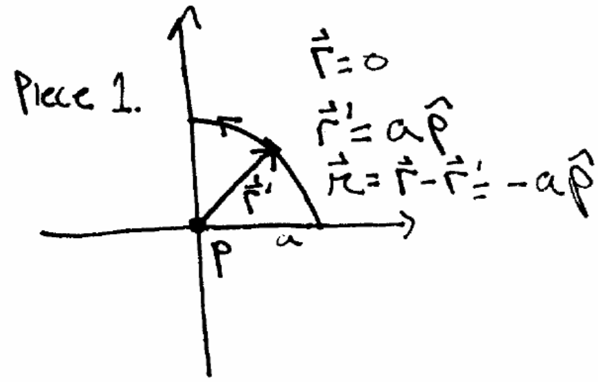
$$= \frac{\mu_0 I\sqrt{2}}{4\pi R}\hat{z}$$

$$\boxed{\vec{B}_{total} = 4\cdot\vec{B}_1 = \frac{\mu_0 I\sqrt{2}}{\pi R}\hat{z}}$$

Griffiths 5.9



$\Rightarrow$



$$\vec{B}(r) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

since $d\vec{l}$ and $\vec{\mathscr{r}}$ in same direction for pieces 2. and 4. $d\vec{l} \times \vec{\mathscr{r}} = 0$ for these pieces $\rightarrow$ only 1. and 3. contribute

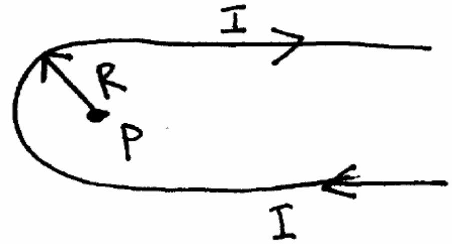
Due to symmetry of the problem, let's choose cylindrical coords. So $d\vec{l} = a\,d\phi\,\hat{\phi}$ for piece 1.
$d\vec{l} = -b\,d\phi\,\hat{\phi}$ for piece 3.

$$\underset{r=0}{B(0)} = \frac{\mu_0 I}{4\pi} \int_{1.} \frac{a\,d\phi\,\hat{\phi} \times -a\hat{\rho}}{a^3} = \frac{\mu_0 I}{4\pi} \int_0^{\frac{\pi}{2}} \frac{+a^2\,d\phi}{a^3}\hat{z} = +\frac{\mu_0 I}{8a}\hat{z}$$

piece 2 just gives $-\dfrac{\mu_0 I}{8b}\hat{z}$

$$B(0) = 1. + 2. = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)\hat{z}$$

5.9 b)



$\vec{B}$ field at P = two semi infinite wires + half circle

= one infinite wire + half circle

from 5.8 a) $\vec{B}$ field from a wire segment

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{R \, \hat{z}}{(R^2+y^2)^{3/2}} \, dy = \frac{\mu_0}{4\pi} I \frac{1}{\sqrt{R^2+y^2}} \hat{z}$$

in limits $[0, \infty]$ $\quad \vec{B} = \frac{\mu_0 I}{4\pi R} \hat{z}$

for two semi-infinite segments or one fully infinite

$$\boxed{\vec{B} = 2 \cdot \frac{\mu_0 I}{4\pi R} \hat{z} = -\frac{\mu_0 I}{2\pi R} \hat{z}}$$

two-infinite wires

Half-circle

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$d\vec{l} = R \, d\phi \, \hat{\phi}$$

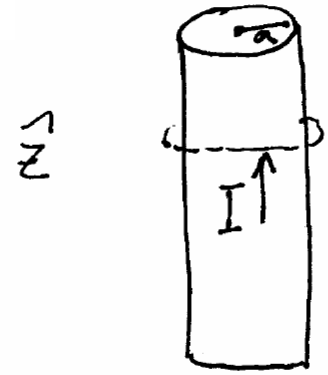
$$\mathscr{r} = -R \hat{r} \qquad \vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{\pi} \frac{R^2 \hat{z} \, d\phi}{R^3}$$

$$\boxed{\vec{B} = -\frac{\mu_0}{4R} \hat{z}}$$ half-circle

$$\boxed{\vec{B}_{total} = -\frac{\mu_0 I}{4R}\left(1 + \frac{2}{\pi}\right) \hat{z}}$$

- sign denotes direction into page

Griffiths 5.13



a) current dist. just on outside surface

Amperian loop radius s

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \qquad \vec{B} = B\hat{\phi} \text{ Right hand rule}$$

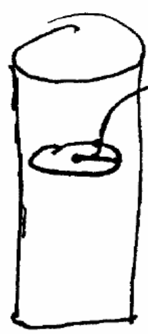
$$\vec{B} = \frac{\mu_0 I_{enc}}{2\pi s}\hat{\phi}$$ any cylinder ($\vec{I}$ flowing along length)

for $s<a$ $\quad I_{enc}=0 \quad \vec{B}=0$

$s>a \quad I_{enc}=I \quad \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

b) $\vec{J} = ks$ where k const

$$\vec{B} = \frac{\mu_0 I_{enc}}{2\pi s}\hat{\phi} \qquad s<a \qquad I_{enc} = \int \vec{J}\cdot d\vec{a}$$



Amperian loop radius s

$$= \int_0^s\int_0^{2\pi} ks\, s\,d\phi\,ds$$

$$= \frac{2\pi k s^3}{3}$$

$$\boxed{\vec{B} = \frac{\mu_0 k s^2}{3}\hat{\phi}}$$

if you want

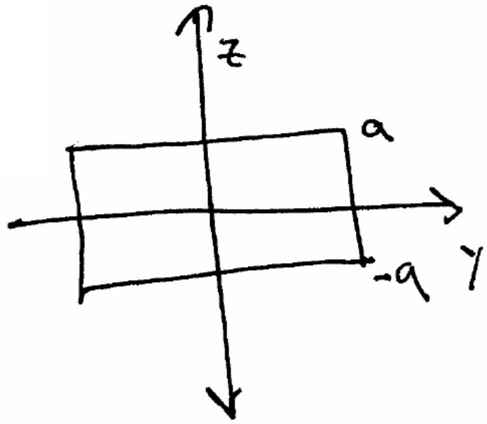
$$I = \int_0^a\int_0^{2\pi} \vec{J}\cdot d\vec{a}$$

or $\quad = \frac{2}{3}\pi k a^3$

so $k = \frac{3I}{2\pi a^3}$ $\quad \boxed{\vec{B} = \frac{\mu_0 I s^2}{2\pi a^3}}$

$$\boxed{s>a \quad \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

Griffiths 5.14



cross section of infinite slab

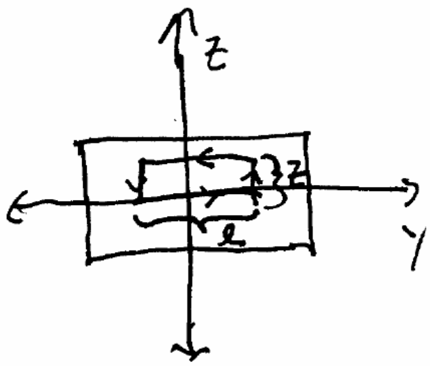
$\vec{J} = J\hat{x}$ uniform current density

$$\left.\begin{array}{ll} \vec{B} = -B\hat{y} & z>0 \\ \vec{B} = B\hat{y} & z<0 \\ B = 0 & \text{when } z=0 \end{array}\right\}$$

Not the most intuitive.

See ex. 5.8

Inside Slab



not $2l$ since only top of loop contributes

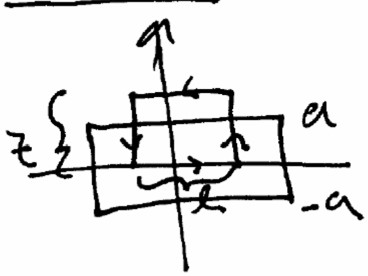
$$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow Bl = \mu_0 I_{enc}$$

$$\vec{B} = -\frac{\mu_0 I_{enc}}{l}$$

$$I_{enc} = \int \vec{J}\cdot d\vec{a} = J\int_0^z\int_0^l dy\,dz = Jzl$$

$$\vec{B} = -\mu_0 J z\,\hat{y}$$

Outside



Again $\vec{B} = -\frac{\mu_0 I_{enc}}{l}\hat{y}$

$$I_{enc} = \int_0^a\int_0^l dy\,dz = Jla$$

$$\boxed{\begin{array}{ll} \vec{B} = -\mu_0 J a\,\hat{y} & z>0 \\ \quad\;\; \mu_0 J a\,\hat{y} & z<0 \end{array}}$$

since no z-dep sign gets lost need two cases