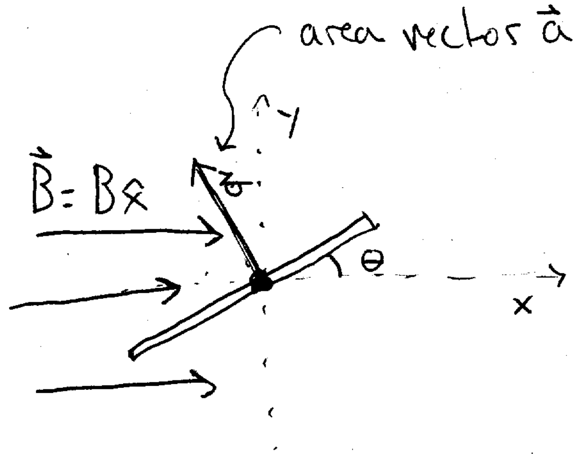
HW7 Soln

1. Griffiths 7.10



$\vec{\omega} = \omega\hat{\phi}$

At any given moment $\vec{B}\cdot d\vec{a} = -B\sin\theta\, dx\,dy$

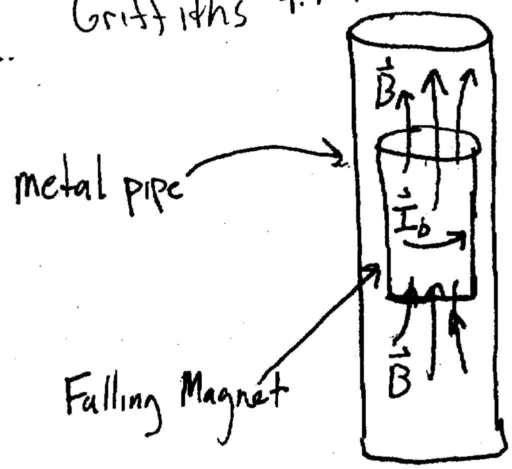
$\theta = \omega t$

side of square

$$\Phi_B \int_0^a\int_0^a \vec{B}\cdot d\vec{a} = -Ba^2\sin(\omega t)$$

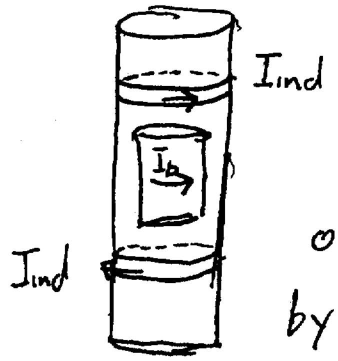
$$\mathcal{E}(t) = -\frac{\partial \Phi_B}{\partial t} = +\omega B a^2\cos(\omega t)$$

2. Griffiths 7.14



- Magnet inside the cylinder produces a $\vec{B}$ field along $\hat{z}$. $\vec{M}$ is along $\hat{z}$, so the induced bound current $\vec{I}_b$ is in $\hat{\phi}$

- Looking at a strip of pipe <u>Above</u> the magnet, the flux decreases as the magnet falls. Nature opposes this flux by producing its own in the z direction.



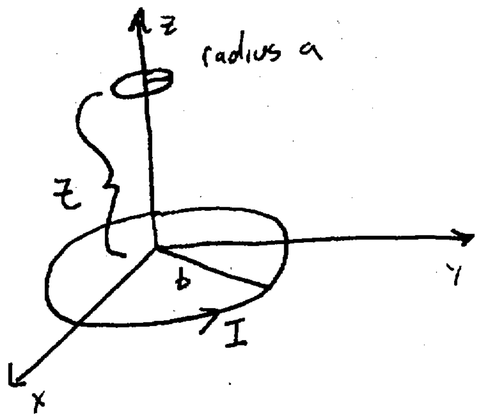
By pointing your thumb in this direction (nature's flux) your fingers curl in the direction of the induced current.

- Looking at a strip of pipe <u>below</u> the magnet, we find an increasing flux in the $+\hat{z}$ direction. Nature's flux opposes this in the $-\hat{z}$ direction, and tells us the induced current is opposite to that above.

(2 cont.)

since like currents attract and opposite currents repel, the induced fields create a force that opposes gravity. This is why it takes some
on the magnet
time for the magnet to reach the bottom.

3. 7.20 Griffiths



a) since $a \ll z$ $\vec{B}$ through small loop $\simeq$ const

From Griffiths Ex 5.6 or your mid term problem 5.

$$\vec{B}_{loop}(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}} \hat{z} = \frac{\mu_0 I}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \hat{z}$$

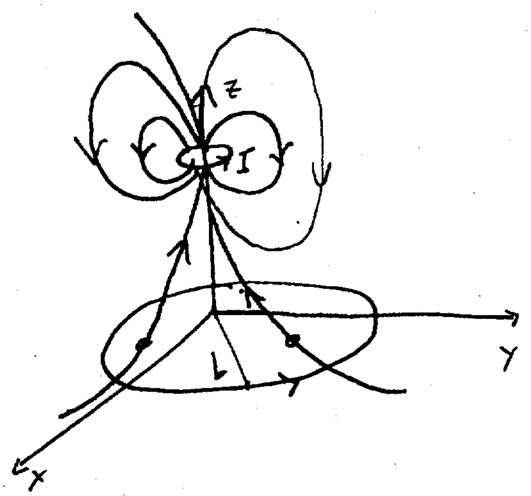
3a) cont.

$$\Phi_{loop2} = \int \vec{B}_1 \cdot d\vec{a}_2 = \int_0^{2\pi}\int_0^{a} \frac{\mu_0 I}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \hat{z} \cdot \rho\, d\rho\, d\phi\, \hat{z}$$

$$= \frac{\mu_0 I}{2} \frac{b^2}{(b^2+z^2)^{3/2}} \int_0^{2\pi}\int_0^{a} \rho\, d\rho\, d\phi$$

$$= \frac{\mu_0 I}{2} \frac{b^2 \pi a^2}{(b^2+z^2)^{3/2}}$$

b)



little trickier

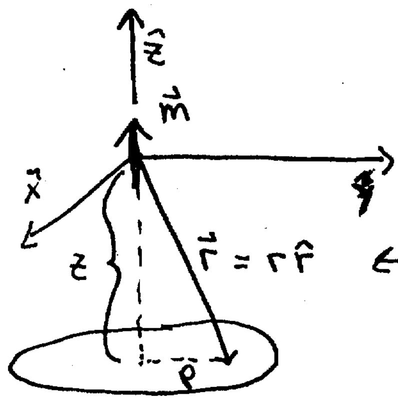
$$\Phi_{loop1} = \int \vec{B}_2 \cdot d\vec{a}_1$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right] \qquad d\vec{a}_1 = \rho\, d\rho\, d\phi\, \hat{z}$$

$$\vec{m} = I\pi a^2 \hat{z} \qquad \hat{r} \text{ cylindrical} = \frac{\rho\hat{\rho} + -z\hat{z}}{(\rho^2+z^2)^{1/2}}$$

z is negative since we place origin at $\vec{m}$ (see pg 4)

(2b cont)



← this is the r and $\hat{r}$ in $\vec{B}_2$

$$r = (z^2 + \rho^2)^{1/2}$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

$$\vec{m} = I\pi a^2 \hat{z} \qquad \hat{r} = \frac{\rho\hat{\rho} - z\hat{z}}{(\rho^2 + z^2)^{1/2}}$$

$$\vec{B}_2 = \frac{\mu_0}{4\pi r^3}\left[\frac{-3mz(\rho\hat{\rho} - z\hat{z})}{(\rho^2 + z^2)} - m\hat{z}\right]$$

Plug in $(\rho^2 + z^2)^{1/2}$

$$= \frac{\mu_0 I \pi a^2}{4\pi}\left\{\frac{3z(z\hat{z} - \rho\hat{\rho})}{(\rho^2 + z^2)^{5/2}} - \frac{\hat{z}}{(\rho^2 + z^2)^{3/2}}\right\}$$

$$\Phi_{loop\,1} = \int_0^{2\pi}\int_0^{b} \frac{\mu_0 I \pi a^2}{4\pi}\left[\frac{3z^2}{(\rho^2 + z^2)^{5/2}} - \frac{1}{(\rho^2 + z^2)^{3/2}}\right]\rho\, d\rho\, d\phi\, \hat{z}$$

$$\Phi_{loop\,1} = \frac{\mu_0 I \pi a^2}{2} \int_0^b \left[\frac{3z^2}{(\rho^2+z^2)^{5/2}} - \frac{1}{(\rho^2+z^2)^{3/2}} \right] \rho\, d\rho\, \hat{z}$$

$$= \frac{\mu_0 I \pi a^2}{2} \left[-\frac{z^2}{(\rho^2+z^2)^{3/2}} + \frac{1}{(\rho^2+z^2)^{1/2}} \right] \Bigg|_0^b \hat{z}$$

$$= \frac{\mu_0 I \pi a^2}{2} \left(\frac{-z^2+\rho^2+z^2}{(\rho^2+z^2)^{3/2}} \right) \Bigg|_0^b \vec{z}$$

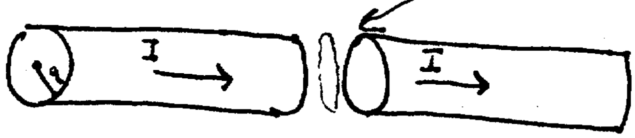
$$= \frac{\mu_0 I \pi a^2 b^2}{2(b^2+z^2)^{3/2}} \hat{z} \quad \checkmark$$

c) dividing by I $M_{12} = M_{21}$

since $\Phi_1 = M_{12} I$, $\Phi_2 = M_{21} I$

from a) and b) $\Phi_1 = \Phi_2$

Griffiths 7.31



draw Amperian loop inside gap

any current in the gap would have to be displacement current I_d

$$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{d\,enc} \qquad \vec{B} = \frac{\mu_0 I_{d\,enc}}{2\pi\rho}\,\hat{\phi}$$

from pg 324, $\dfrac{\varepsilon_0 \partial E}{\partial t} = \dfrac{I}{A}$ of charging capacitor

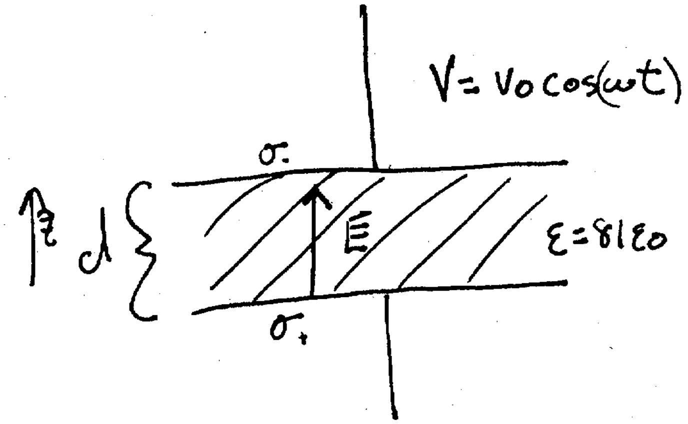
$$J_d = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{I}{A} = \frac{I}{\pi a^2}$$

$$I_d = \int_0^\rho J_d \cdot da = \frac{I}{\pi a^2}\pi\rho^2 = I\frac{\rho^2}{a^2}$$

$$\vec{B} = \frac{\mu_0 I_{d\,enc}}{2\pi\rho}\,\hat{\phi} = \boxed{\frac{\mu_0 I \rho}{2\pi a^2}\,\hat{\phi}}$$

If you read the problem carefully and technically, it doesn't say capacitor is <u>charging</u>. If you put $\vec{B} = 0$, technically this is right.

Griffiths 7.37



$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}, \quad V = -\int_d^0 \vec{E}\cdot d\vec{l} = \frac{\sigma d}{\varepsilon_0}$$

$$E = \frac{V}{d}$$

$$J_c = \sigma E = \sigma \frac{V}{d} = \frac{\sigma V_0 \cos(\omega t)}{d}$$

$$J_d = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = -\frac{\varepsilon \omega V_0 \sin(\omega t)}{d}$$

$$\frac{J_c}{J_d} = -\frac{\sigma \cancel{V_0}\ \cancel{d}}{\cancel{d}\ \varepsilon\omega \cancel{V_0}} = \frac{\sigma}{\varepsilon\omega} = \frac{1}{2\pi\nu\varepsilon\rho}$$

Just interested in ratio of Amplitudes think of as RMS Average if you like

resistivity

$$= \frac{1}{2\pi(4\times10^8\,Hz)(81)(8.85\times10^{-12}\,\frac{C^2}{Nm^2})} \cdot \frac{1}{.23\,\Omega\cdot m}$$

$$\boxed{= 2.41}$$

6. a) $V = -L\dfrac{dI}{dt}$ just Faradays law

$V = -\dfrac{\partial \Phi_{self}}{\partial t}$ $\Phi_{self} = LI$

self inductance

$\mathcal{E} = -\dfrac{\partial \Phi_{self}}{\partial t}$

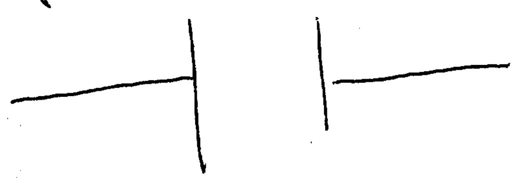
$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

$$\int (\nabla \times \vec{E})\cdot d\vec{a} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \checkmark$$

displacement current

$I = c\dfrac{dV}{dt}$ voltage across cap

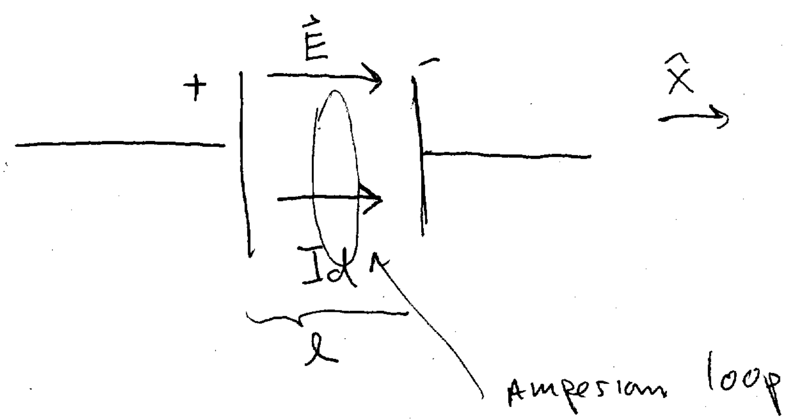


$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

6 as cont.

$$\int (\nabla \times B) \cdot d\vec{a} = \int \mu_0 \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{J_d} \cdot d\vec{a}$$

$$\Rightarrow \oint B \cdot dl = \mu_0 I_{d\,enc}$$



$$\mu_0 \frac{\partial}{\partial t} \int \varepsilon_0 \vec{E} \cdot d\vec{a} = \mu_0 I \qquad I = I_{d\,enc}$$

$$\varepsilon_0 = \frac{C}{l} \leftarrow \text{length} \qquad \vec{E}_{\parallel \text{plate cap}} = \frac{\sigma}{\varepsilon_0} \hat{x}$$

$$V = \frac{\sigma}{\varepsilon_0} l \qquad \vec{E} = \frac{V}{l} \hat{x}$$

so $$\mu_0 I = \mu_0 \frac{\partial}{\partial t} \frac{C}{l} \int \frac{V}{l} \cdot da$$

$$I = \frac{C}{l^2} A \frac{dV}{dt} \Rightarrow I = C \frac{dV}{dt} \qquad \frac{A}{l^2} \text{ absorbed into } C$$

6. b)

$$Z_C = \frac{1}{i\omega C}$$

$\omega \uparrow$ $Z_C \downarrow$

Cap likes high freq

$\omega = 0$ $Z_C = \infty$

DC blocked

$$Z_L = i\omega L$$

$\omega \uparrow$ $Z_C \uparrow$

inductor blocks high freq

$\omega = 0$ $Z_C = 0$

looks like a short
no impedance