HW 8 Soln

1. Griffiths 9.11

$$f = A\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_a) \qquad g = B\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_B)$$

$$\tilde{f} = Ae^{i(\vec{k}\cdot\vec{r} - \omega t + \delta_a)} \qquad \tilde{g} = Be^{i(\vec{k}\cdot\vec{r} - \omega t + \delta_B)}$$

$$\tilde{f}\cdot\tilde{g}^* = Ae^{i(\vec{k}\cdot\vec{r} - \omega t + \delta_a)}Be^{-i(\vec{k}\cdot\vec{r} - \omega t + \delta_B)}$$

$$= AB\,e^{i(\delta_A - \delta_B)}$$

RHS in 9.11 $\Longrightarrow$ $\frac{1}{2}AB\cos(\delta_A - \delta_B)$

$$\langle fg\rangle = \frac{1}{T}\int_0^T \underbrace{AB\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_A)\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_B)}\,dt$$

let $x = \omega t$
$y = \vec{k}\cdot\vec{r} + \delta_A$
$z = \vec{k}\cdot\vec{r} + \delta_B$

$$\underbrace{\cos(y-x)\cos(z-x)}$$

$$[\cos(y)\cos(x) + \sin(y)\sin(x)][\cos(z)\cos(x) + \sin(z)\sin(x)]$$

$$\Rightarrow \cos(y)\cos(z)\cos^2(x) + \cos(y)\cos(x)\sin(x)\sin(z) + \sin(y)\sin(x)\cos(x)\cos(z) + \sin(y)\sin(z)\sin^2(x)$$

(1 cont)

$$\frac{1}{T}\int_0^T \sin(\omega t)\cos(\omega t)\,dt = 0$$

$$\frac{1}{T}\int_0^T \sin^2(\omega t)\,dt = \frac{1}{T}\int_0^T \cos^2(\omega t) = \frac{1}{2}$$

so $\langle f \cdot g \rangle = AB\frac{1}{2}\cos(\vec{k}\cdot\vec{r}+\delta_A)\cos(\vec{k}\cdot\vec{r}+\delta_B)$

$\qquad + \frac{1}{2}\sin(\vec{k}\cdot\vec{r}+\delta_A)\sin(\vec{k}\cdot\vec{r}+\delta_B)$

$= \frac{1}{2}AB\cos(\delta_A - \delta_B)$

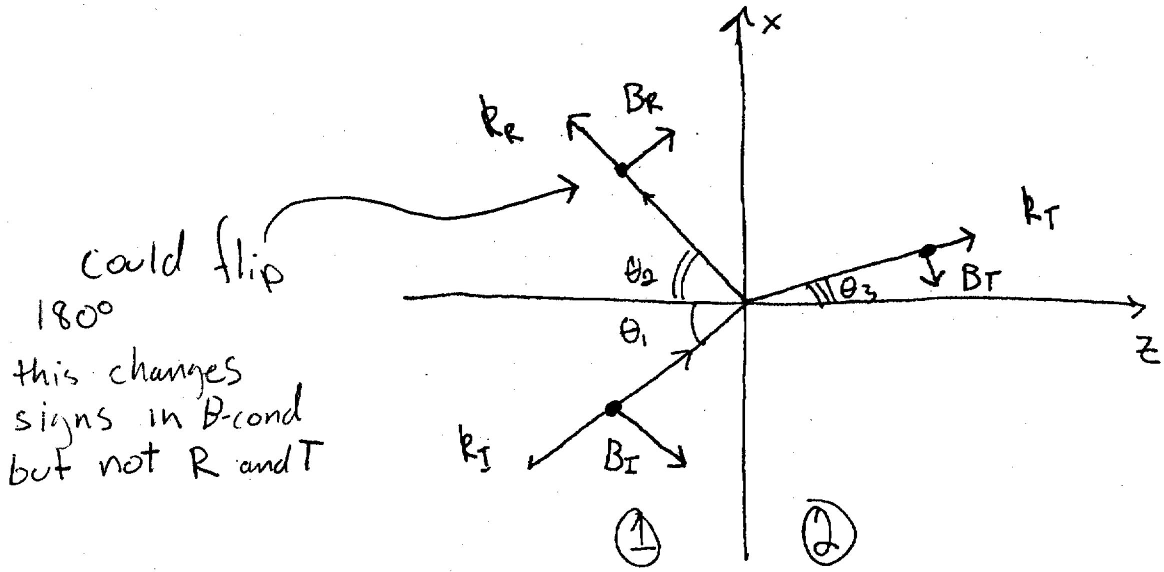
from sum-diff formula

$\cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$

2. Griffiths 9.16 light

see pg 3

2. Griffiths 9.16

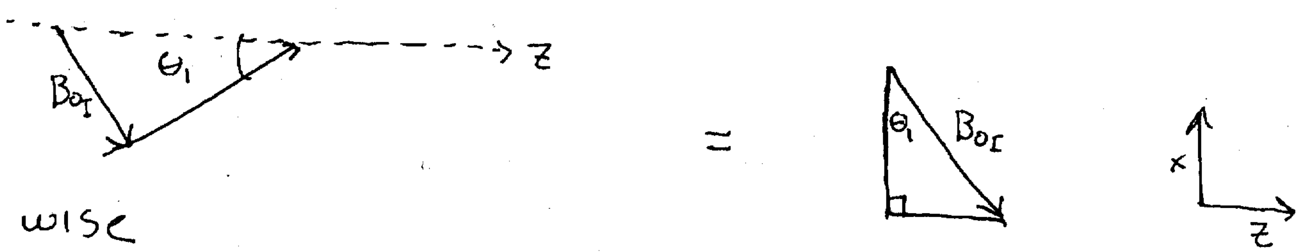


Let $\vec{E}$ be polarized in $\hat{y}$, $\vec{B}$ in the xz plane

a)

$$\vec{E}_I = E_{0I}\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)}\, \hat{y}$$

$$\vec{B}_I = \frac{E_{0I}}{v_1} e^{i(\vec{k}_R\cdot\vec{r} - \omega t)} (-\cos\theta_1 \hat{x} + \sin\theta_1 \hat{z})$$



Like wise

$$\vec{E}_R = E_{0R}\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)}\, \hat{y}$$

$$\vec{B}_{0R} = \frac{E_{0R}}{v_1} e^{i(\vec{k}_R\cdot\vec{r} - \omega t)} (\cos\theta_2 \hat{x} + \sin\theta_2 \hat{z})$$

Finally

$$\vec{E}_T = E_{oT} e^{i(\vec{k}_T \cdot \vec{r} - \omega t)} \hat{y}$$

$$\vec{B}_T = \frac{E_{oT}}{v_2} e^{i(\vec{k}_T \cdot \vec{r} - \omega t)} \left(-\cos\theta_3 \hat{x} + \sin\theta_3 \hat{z}\right)$$

b) i) $\varepsilon_1 E_1^\perp = \varepsilon_2 E_2^\perp$

$\perp$ z comp

$\parallel$ x, y comp

$\Rightarrow \boxed{0 = 0}$

ii) $B_1^\perp = B_2^\perp \Rightarrow \boxed{\frac{1}{v_1} E_{oI} \sin\theta_1 + \frac{1}{v_1} E_{oR} \sin\theta_2 = \frac{1}{v_2} E_{oT} \sin\theta_3}$

iii) $E_1^\parallel = E_2^\parallel \Rightarrow \boxed{E_{oI} + E_{oR} = E_{oT}}$

iv) $\frac{1}{\mu_1} B_1^\parallel = \frac{1}{\mu_2} B_2^\parallel \Rightarrow \boxed{\frac{1}{\mu_1}\left(\frac{1}{v_1} E_o(-\cos\theta_1) + \frac{1}{v_1} E_{oR}(\cos\theta_2)\right) = \frac{1}{\mu_2}\frac{1}{v_2} E_{oT}(-\cos\theta_3)}$

c) $\alpha\beta = \beta \frac{\sqrt{1 - \frac{\sin^2\theta_1}{\beta^2}}}{\cos\theta_1}$, $\beta = 1.5$ $\alpha\beta = \frac{\sqrt{2.25 - \sin^2\theta_1}}{\cos\theta}$

$\alpha\beta \neq 1$ no Brewster

3. Want to know z where

$$e^{-Kz} = \frac{E}{E_0} = .01$$

decaying part

$$-Kz = \ln(.01)$$

$$z = -\frac{\ln(.01)}{K}$$

$$K = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]^{1/2}$$

@ X-ray $\omega \sim 10^{18}\,\frac{rad}{s}$

Copper $\varepsilon = \varepsilon_0 = 8.85\times10^{-12}\,C^2/N\cdot m^2$

$$\mu = \mu_0(1+\chi_m) = 4\pi\times10^{-7}\,N/A^2\,(1-9.7\times10^{-6})$$

$$\simeq 4\pi\times10^{-7}\,N/A^2$$

$$\sigma = 5.95\times10^{7}\,\frac{1}{m\cdot\Omega}$$

$$K = 10^{18}\,\frac{rad}{s}\sqrt{\frac{8.85\times10^{-12}\cancel{C^2}\;4\pi\times10^{-7}\,\cancel{N}\,s^2}{2\,\cancel{N}\cdot m^2\;\cancel{A^2}}}\left[\sqrt{1+(6.72)^2}-1\right]^{1/2}$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{J_c}{J_d} = \frac{5.95\times10^{7}\;N\cdot m^2}{10^{18}\frac{rad}{s}\cdot 8.85\times10^{-12}\,C^2\,m\cdot\Omega} = 6.72$$

$$\Omega = \frac{J\cdot s}{C^2}$$

3 cont.

$$K = 10^{18} \frac{\text{rad}}{s} \frac{s}{m} \sqrt{\frac{8.85\times10^{-12} \cdot 4\pi\times10^{-7}}{2} 2.41}$$

$$= 10^{18} \frac{\text{rad}}{m} (2.346\times10^{-9})$$

$$= 5.68\times10^{9} \ m^{-1}$$

$$z = \frac{-\ln(.01)}{5.68\times10^{9}} m = .35 \ nm$$

Turns out this model not so good when $\omega \gtrsim$ U.V.. Then we need a diff model. See plasma frequency in Wangsness or online. This model shows how metals become transparent for $\omega \gtrsim$ UV. It takes into account atomic nature of the metal. Damn that superman!

3. cont.

For cell phone $\omega \sim 10^9 \frac{\text{rad}}{s}$

so $\frac{\sigma}{\omega \varepsilon} = 6.723 \times 10^9$

$$K = 10^9 \frac{\text{rad}}{s} \frac{S}{m} \left(2.346 \times 10^{-9} \right)$$

$$= 1.93 \times 10^5 \text{ m}^{-1}$$

$$z = 10.34 \ \mu\text{m}$$

Note that when $\frac{\omega \varepsilon}{\sigma} \ll 1$ "good cond"

$K \simeq \sqrt{\frac{\mu \sigma \omega}{2}}$

This is why copper cable doesn't work so well to transmit high-speed signals

solution: Fiber optic transmission!

2009 Nobel Prize

4. Griffiths 9.21

$$R = \left|\left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\right|^2$$ where $\tilde{\beta}$ given by Griffiths as $\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2 \omega} \tilde{k}$

complex prop. vector

$\tilde{k} = \alpha + i\beta$ wangsness notation

first $\mu_1 \approx \mu_2$ so $\tilde{\beta} = \frac{v_1}{\omega}\tilde{k} = \frac{v_1}{\omega}(\alpha + i\beta)$

for a good conductor $\alpha = \beta = \left(\frac{1}{2}\mu\sigma\omega\right)^{1/2}$

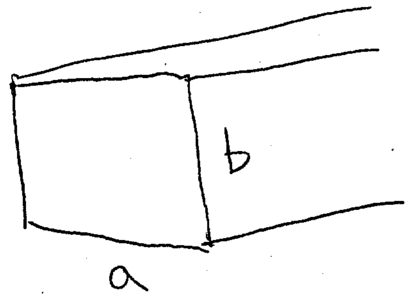
So $\tilde{\beta} \approx \frac{v_1}{\omega}\alpha(1+i)$ Let $A = \frac{v_1\alpha}{\omega}$

$$R = \left|\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right|^2 = \left|\frac{1-A(1+i)}{1+A(1+i)}\right|^2 = \left|\frac{(1-A)+iA}{(1+A)+iA}\right|^2$$

$$= \frac{(1-A)^2 + A^2}{(1+A^2) + A^2} = .92 \quad \text{or } 92\%$$

$$A = \frac{1}{\omega}\sqrt{\frac{\mu_2\sigma\omega}{\mu_1\varepsilon_1 2}} = \sqrt{\frac{10^7 (\Omega \cdot m)^{-1}}{2\left(8.85\times10^{-12}\frac{C^2}{Nm^2}\right)10^{15}\frac{rad}{s}}} \approx 24$$

5. Griffiths 9.28



waveguide

$a = 2.28\,cm \quad b = 1.01\,cm$

$$\nu_{10} = \frac{1}{2\pi}\omega_{10} = \frac{c}{2a}$$ hmm looks like long. modes in a laser

$$= 0.66 \times 10^{10}\,Hz$$

$\nu < .66 \times 10^{10}\,Hz$ cannot propagate

$$\nu_{20} = \frac{2c}{2a} = 1.32 \times 10^{10}\,Hz$$

microwave

to get just <u>one</u> mode $\quad 0.66 \times 10^{10} < \nu < 1.32 \times 10^{10}\,Hz$

$$\lambda = \frac{c}{\nu}, \quad \lambda_{10} = 2a$$

$$\lambda_{20} = a$$

$$2.28 < \lambda < 4.56$$