8/10

Nuclear Magnetic Resonance

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We proposed to measure the gyromagnetic ratio, γ, and the g-factor for protons using nuclear magnetic resonance techniques. We made our measurement by inducing spin-flip energy transitions in a sample of protons, $\Delta E = h\nu_f = h\gamma B/2\pi$, where frequency, ν, and magnetic field, B, corresponded to values measured at resonance. By inducing resonance, measuring the corresponding magnetic field, and frequency, we were able to obtain a value for the gyromagnetic ratio for protons, $\gamma = 2.616 \times 10^8\ s^{-1}T^{-1}$ +/- $1.522 \times 10^8\ s^{-1}T^{-1}$. The g-factor, given by $g = \gamma h/2\pi\mu_N$, where μ_N is the nuclear magneton, we measured to be g = 5.462 +/- 0.505.

I. Introduction

In 1952, Felix Bloch and Edward Purcell (From Stanford University and Harvard University respectively) shared the Nobel prize for observing nuclear magnetic resonance (NMR) in bulk material. The drive for conducting such experiments began with the attempt to understand the structure and dynamics of nuclei. In 1924, Pauli proposed that nuclei might contain angular momentum, as well as a magnetic moment, in attempt to explain hyper-fine splitting of energy levels observed in atomic spectra. Optical techniques at this time were not refined enough to verify these hypotheses[i]. However, in 1939, I. Rabi developed such an instrument. Rabi performed the first nuclear magnetic resonance experiment, for which he won the Nobel prize in 1944, and with his instrument, determined the magnetic moment of the proton and deuteron, as well as measured the Lamb shift[ii]. Rabi's experiment consisted of sending a beam of atoms through a system of magnets. Further adaptation of Rabi's experiment by Purcell and Bloch, demonstrated that NMR could be performed on bulk material. Such techniques have led to the use of NMR for determining the physical and chemical structure of different materials. However, the most popular current use of NMR technology has been the non-evasive technique of using NMR coupled with imaging technology to produce images of the inside of human body, by the medical profession.

The basic principles of NMR can be explained both classically and quantum mechanically. From quantum mechanics, we know that when a constant magnetic field is applied to a free hydrogen atom, the acceptable eigen-energies for the system are $E_o + E'$, where E_o is the energy of the unperturbed atom, and E' is the energy of determined from the perturbation. In this case, $E' = -\mu B_z$, where μ is the magnetic moment of the proton, and B is the perturbing magnetic field (for our purposes, assume the field is in the z direction). This is known as the Zeeman effect, and leads to a splitting in the energy of the free hydrogen atom, as $\mu = \gamma m_s h/2\pi$, where m_s is the spin equal to +/- ½ for a proton. The difference in energy of the two spin states then is just $\Delta E = h\gamma B_z/2\pi$ (see Figure 1.). If we allow another perturbing magnetic field, this one rotating at a frequency ω_f and normal to the original perturbation field (pointing in the rho direction), transitions from spin $-1/2$ to spin $+1/2$ (and visa-versa) will occur when $h\omega_f/2\pi = \Delta E$, or $\omega_f = \gamma B_z$. Transitions occur only when the energy of the perturbing field is equal to the split of levels; that is transitions occur when the frequency of the perturbing field is resonant with the magnetic system (for the explicit math concerning the transition rate, see Melissinos pp. 345-348).

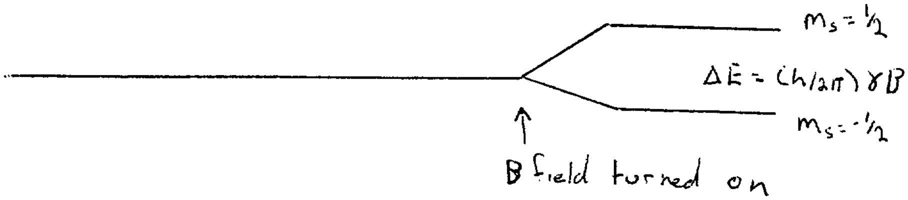


Figure 1.

The classical analog of this phenomena can best be explained in terms perturbations of gyroscopic motion. We imagine proton spinning along the axis of its magnetic moment at an angle theta with respect to the z-axis (see Figure 2.)[iii]. Applying a constant magnetic field along the z-axis causes the electron to precess about the axis at the constant angle theta at the Larmour frequency, $\omega_L = n_z|dJ/dt|/|J \times n_z| = \gamma(J \times B)/|J \times n_z| = \gamma B_z$, (where n_z is the unit vector in the z-direction, and J is the angular momentum of the proton). If another field is applied, this one pointing in the rho direction and oscillating about the z-axis at a frequency ω_f, the spin of the proton will flip when $\omega_f = \omega_L = \gamma B_z$.

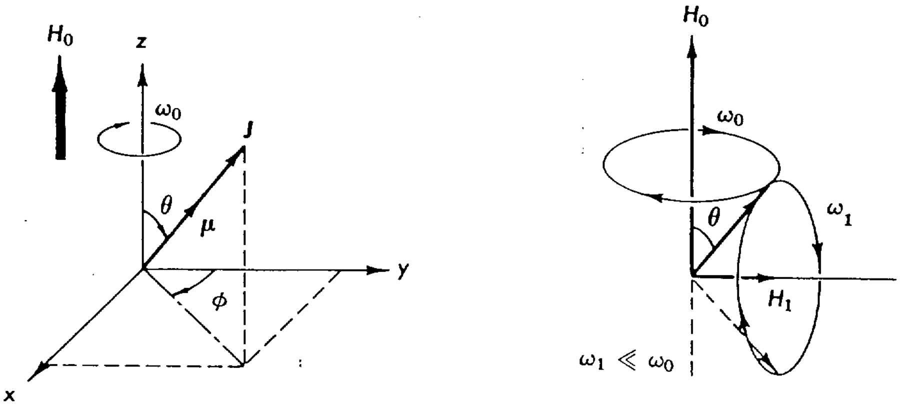


Figure 2.

Whether viewed classically as the frequency of the oscillating magnetic field matching the Lamour frequency, or as the energy of the oscillating field matching the amount of discrete energy required for transitions, the name magnetic resonance is applied to both. One can force resonance to occur by keeping a system of protons in a constant fixed magnetic field (fixing the Lamour frequency), and varying the oscillating field frequency until they match, or by fixing the oscillating field frequency, and varying the constant magnetic field (the Larmour Frequency) until they match. Our experimental technique consisted of the former, the same technique used by Purcell. Our NMR experiment mimicked that of Purcell's in method and Rabi's in purpose. We proposed to measure the gyromagnetic ratio and the g-factor of the proton, constants that comprise its magnetic moment, through the nuclear magnetic resonance of a bulk sample.

II. Experimental Setup

The basic pieces of equipment we used to produce magnetic resonance were a magnetic field source to provide a constant field, an oscillating magnetic field source, and an oscilloscope with which to detect the resonance. Our constant magnetic field source consisted of an electromagnet that produced fields on the order of 10^3 Gauss. The only constraint on the field is that it had to correspond to obtaining matching frequencies in the radio frequency band. However, the signal to noise ratio increases as $\omega_f^{3/2}$, implying that the larger the field, the better signal to noise[iv]. The electromagnet contained a gap large enough in which to wedge the oscillating magnetic field circuit, which also contained the sample (see figure 3.)[v]. We were able to set the frequency of the oscillating field, after which we varied the constant magnetic field until we could view the resonance on the scope.

The detection of the resonance on the scope was made possible by properties of the oscillating field circuit. This circuit (see Figure 4.), known as a marginal oscillator not only produced the r.f. field, but gave up some of its energy to the sample whenever resonance occurred (that is it gave up the amount of energy needed to cause spin-flip energy transitions). As this happened, a resistance was added to the circuit that in turn decreased the amplitude of the r.f. oscillations $(V \sim Q = \omega L/R)$[vi]. This decrease in voltage appeared as a dip on the scope. So, the magnetic field of the electromagnet was varied until a dip was viewed on the scope, whereby resonance was obtained. At this point, the magnetic field of the electromagnet was measured with a Gauss-meter. We proposed that plotting the values of different r.f. frequencies of the oscillating field vs. the corresponding constant magnetic fields at resonance would produce a line whose slope was the gyro-magnetic ratio γ.

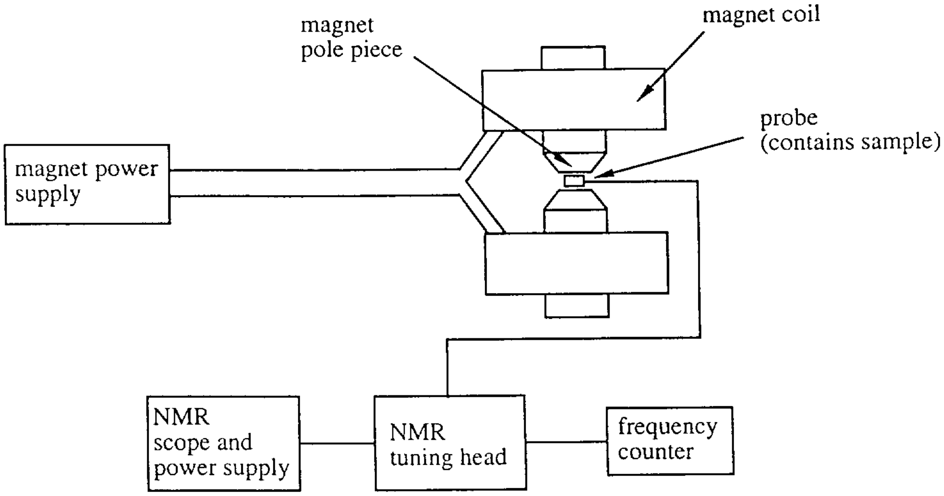
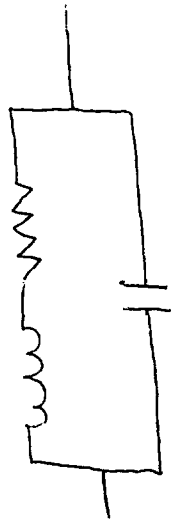


Figure 3.



$V = IZ$

Figure 4.

This, however, does not describe the complete story of the equipment. To aid in the search of resonance dips, a small modulating magnetic field was produced by the r.f. circuit to run parallel with the dc field of the electromagnet. This field which oscillated at 60 Hz, helped to create a range at which resonance could occur. When any part of the superposed ac signal traversed the resonance value for the magnetic field, dips could be viewed on the scope (see Figure 5.)[vii]. Lining up the signal directly on top of the resonance entailed the resonance dip occurring every 120 Hz, that is for every zero of the modulating field.

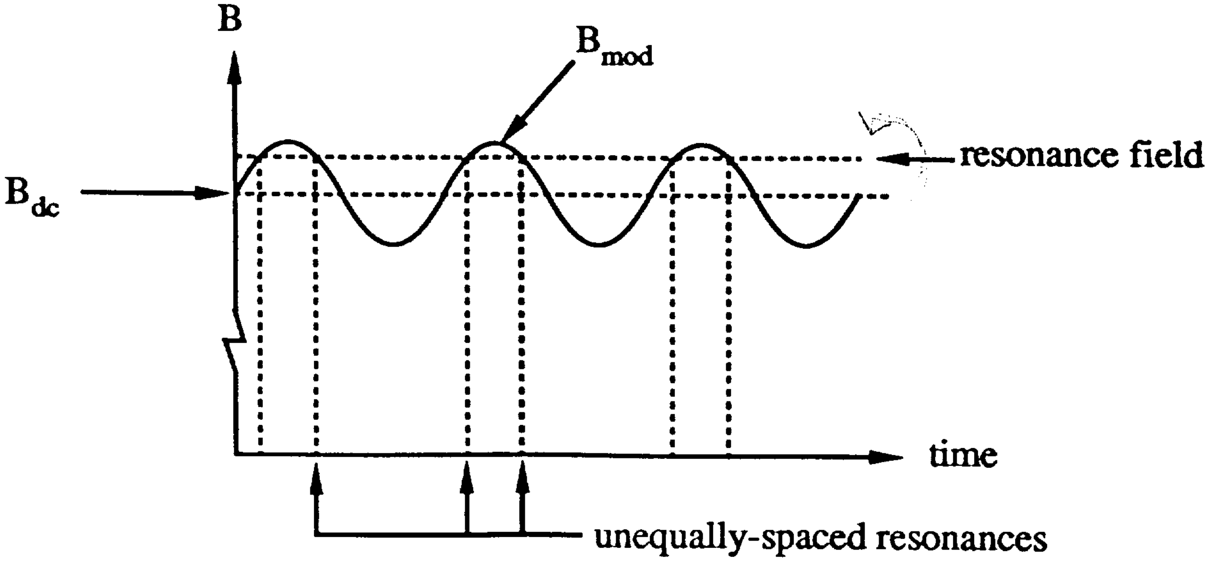


Figure 5.

III. Results

Figure 6 shows a plot of frequency versus magnetic field for two different enclosed proton samples. For each sample, five different r.f. frequencies were chosen, and the corresponding magnetic resonance field was measured and plotted. Figure 7 demonstrated the same type of ~~plot,~~ but for a single ~~sample~~ of mineral oil (rich in Hydrogen atoms- protons). The slopes of each line were 4.138 x 10^7 $s^{-1}T^{-1}$ +/- 2.269 x 10^7 $s^{-1}T^{-1}$ and 4.187 x 10^7 $s^{-1}T^{-1}$ +/- 2.574 x 10^7 $s^{-1}T^{-1}$, as determined my ~~a~~ method of least squares fit, and the standard error approximation for this fitting method (see Appendix of Data and Calculations).

Figure 8 shows the time stream on the scope in a small range near the resonance dip. The decaying series of wiggles after the dip was fit with an exponential function. This was done by taking the natural log of points on the maxima of the decaying wave, plotting them against time, and again linearly fitting by method of least squares (Figure 9.). Assuming the exponential function A = AoexpBt, we take the ln of the entire equation to get $\ln A = \ln A_o + Bt$. If the function is truly exponential, plotting lnA versus t should produce a line whose slope is B and intercept is $\ln A_o$. By fitting this line, B and A_o can be solved, and plugged back into the exponential function. This procedure was accomplished here and produced an A_o of 0.358832 Volts, and a slope B of –2246.55 s^{-1} +/- 160.278. Finally, the frequency of this decaying oscillation was determined to be 4.577 kHz +/- 0.105 kHz.

IV. Discussion

You should put error bars on your data points. The error bars should come from the uncertainty of your measurement!

If we accept the relation $\nu_f = \gamma B_z/2\pi$, then the average gyromagnetic ratio of protons from Figure 6 and 7, the average of the slopes of these plots, is γ = 2.616 x 10^8 $s^{-1}T^{-1}$ +/- 1.522 x 10^8 $s^{-1}T^{-1}$. This is well within the accepted measured value which is γ = 2.675 x 10^8 $s^{-1}T^{-1}$. However, unfortunately the large uncertainty in our value renders it a somewhat useless measurement. The reason for such a large uncertainty is due to low statistics. The error equation for the slope of a fitted line is $\Delta m = \Sigma d_i/\Sigma (x_i - x_{avg})$ n-1, where $d_i = y_i - mx_i - b$ [viii]. The larger n, the better the uncertainty. Unfortunately, nine to ten points were not enough to statistically give us a powerful measurement for our calculation of the slope. From our value of γ, we obtained the g-factor for the proton $g = \gamma h/2\pi\mu_N$ = 5.462 +/- 3.178, compared to an accepted value of 5.586. Again, the uncertainty was very large due to low statistics.

The frequency of the beat should increase with time

Figure 8 demonstrates the beating of the Larmour frequency against the r.f. frequency. This was caused by our slowly modulating field slightly shifting the Larmour frequency (the magnetic field) from the resonance value at which we set it, throughout the modulation cycle. The amplitude of the modulating field was at a maximum of 1% of the D.C. field, that is 9.82 Gauss +/- 0.08 Gauss, for the r.f. frequency of 4.1716 MHz. The Larmour frequency for a field of 982 – 9.82 Gauss, is $\nu_L = \gamma B/2\pi$ = (.097218 T) (2.675 x 10^8 $s^{-1}T^{-1}$)/2π = 4.1390 MHz. Therefore, the predicted maximum beat frequency should be approximately 32.6 kHz, which is consistent with our measured value lf 4.577 kHz +/- 0.105 kHz. The envelope of the beat oscillations fit well to a decaying exponential. This is consistent with theory of spin-spin relaxation. When the spins of the protons flip at resonance, they proceed to interact with each other causing the collapse of the current excited state, they relax. This occurs as an exponential decay $A = A_o e^{-t/T2}$ where T2 is the spin relaxation time constant. This was the inverse of the decay constant 1/B, for which we fit our exponential envelope. We measured T2 to be 1/2246.55 s^{-1} +/- 160.278 or 4.451 x 10^{-4} s +/- 0.317 x 10^{-4}. Such a spin relaxation time constant is consistent with those accepted experimentally.

V. Conclusion

All of our measurements were consistent with accepted values. Through NMR techniques we were able to measure the gyromagnetic ratio of the proton, as well as its g-factor within 3% of their accepted values. However, due to poor statistics, we could not claim our results with a decent amount of certainty.

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[i] www.pma.caltech.edu/~derose/labs/exp5.html

[ii] Melissnos, Adrian C., Experiments in Modern Physics. p. 342

[iii] p. 348
[iv] p. 358
[v] University of Chicago, Lab Manual, The Structure of Matter III. p. 69
[vi] p. 66
[vii] p. 62
[viii] Squires, G.L., Practical Physics, p. 34

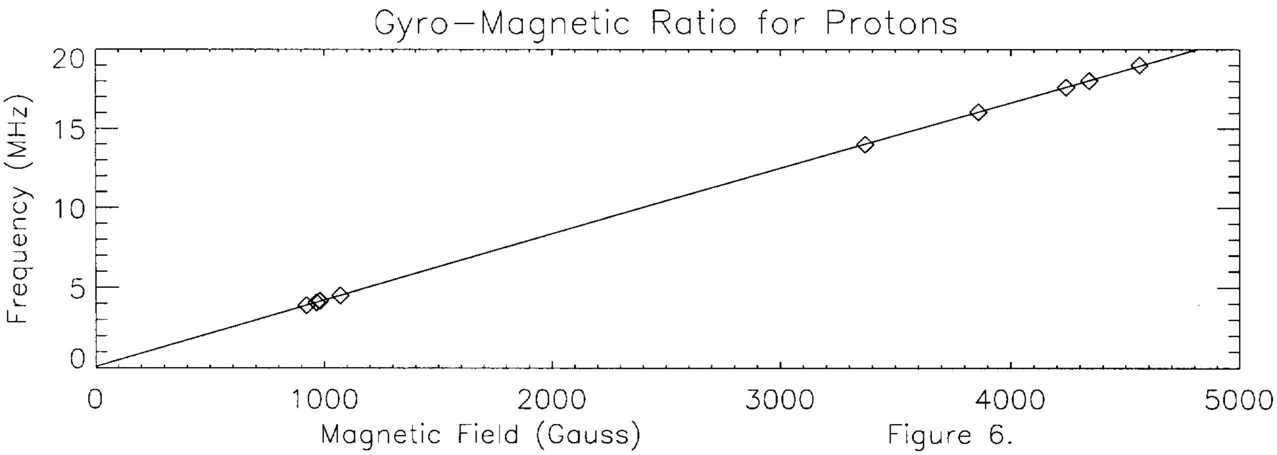

Figure 6.

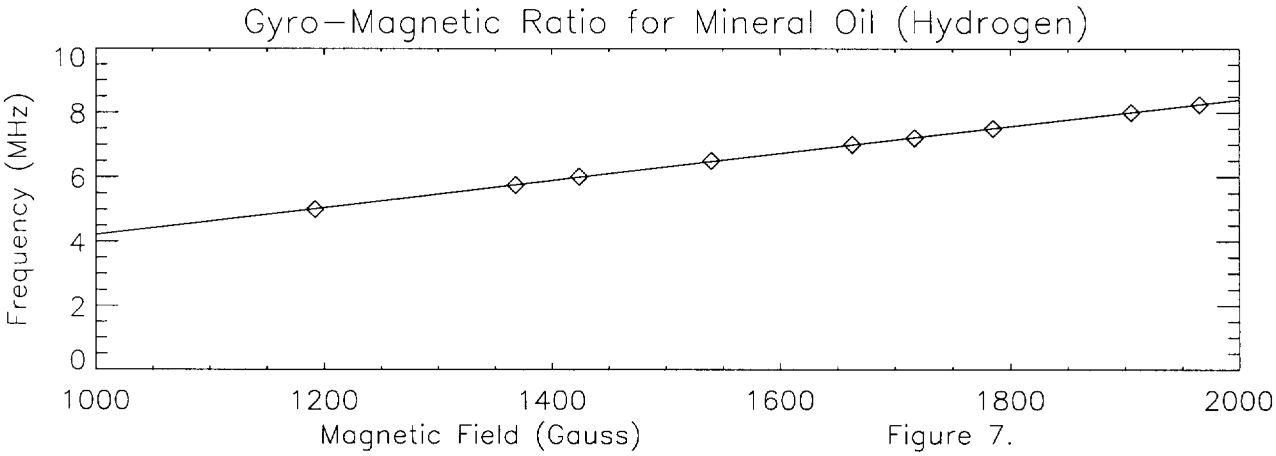

Figure 7.

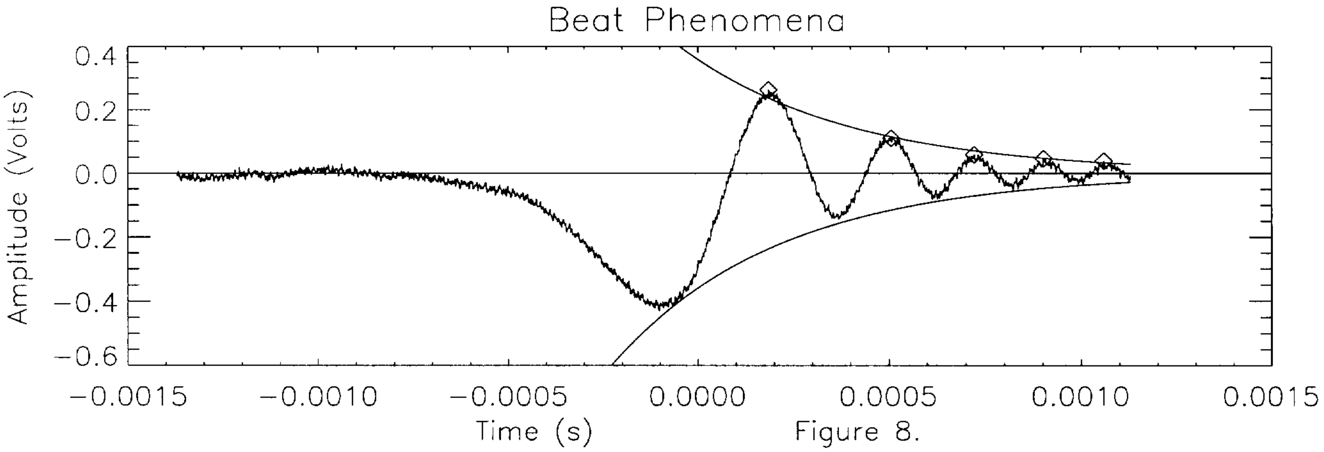

Figure 8.

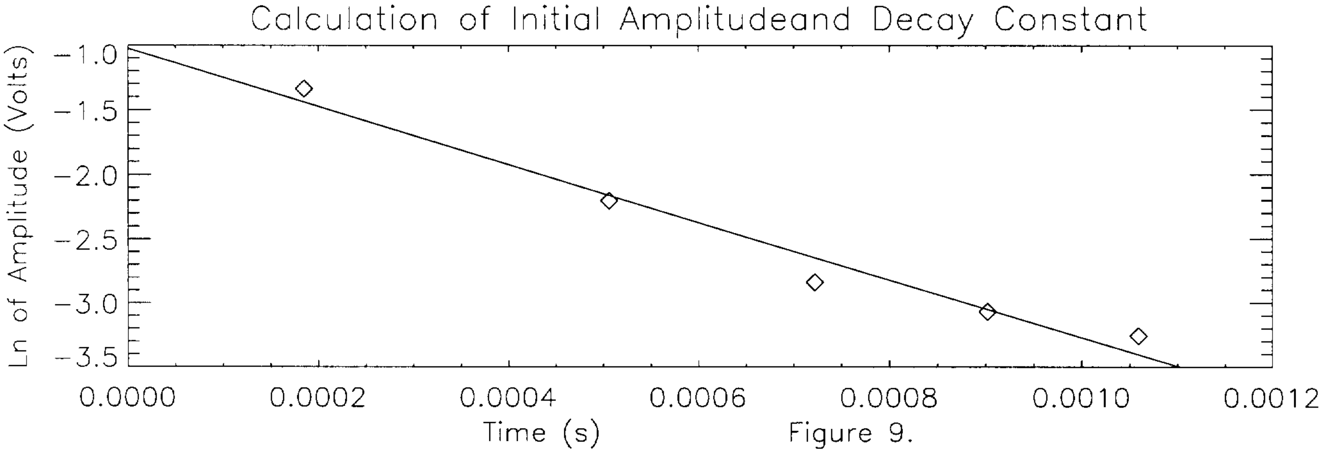

Figure 9.

List of Questions

1. How large are the nuclear energy level splittings at B = 2000 G?

$\Delta E = h\gamma B/2\pi = (6.582 \times 10^{-16}$ eV s$)(2.675 \times 10^{8}$ s^{-1} T$^{-1})(0.2$ T$) = 3.521 \times 10^{-8}$ eV.

2. What fraction of the total number of protons contributed to NMR absorption?

The number of nuclei that can contribute to the absorption of energy is given by the Boltzman distribution:

$N_{abs} = N_{+1/2} - N_{-1/2} = N/2[\, e^{(h\nu/2kT)} - e^{-(h\nu/2kT)}]$, so that the fraction of protons contributing to NMR absorption $N_{abs}/N = 1/2[\, e^{(h\nu/2kT)} - e^{-(h\nu/2kT)}]$. For T = 300 K, and ν = 5.0 MHz (a field of 1174.427 Gauss),

$N_{abs}/N = 8.047 \times 10^{-7}$

3. What do you predict for the temperature dependence of the NMR signal?

As T increases, the fraction of protons contributing to NMR absorption goes down, thereby decreasing signal strength.

4. How does signal strength depend on B, the dc magnetic field?

Changing the dc B field, changes the frequency (energy) entered in the Boltzman distribution. As B increases, so does the frequency, and so does the population of protons contributing to absorption thereby increasing signal strength. However, when enough transitions to the upper state occur, the system will decay toward equilibrium and at an exponential rate; that is the system saturates. However, the interaction between the spins and the lattice tends to restore the Boltzman distribution (Melissinos, 352). T1 is known as the spin-lattice relaxation time, or the time that it takes the system to relax to the equilibrium state.

5. Why is it easier to measure NMR at high magnetic fields than low ones?

The larger the magnetic field, the larger the signal to noise ratio proportional to $\omega^{3/2}$. However, if the field is too large, a larger frequency is required for resonance. Higher frequencies tend to saturate the system, forcing it to decay back to equilibrium at a rate proportional to $e^{-t/T1}$. If T1 is large, it take a long time for the signal to decay back to equilibrium. This seems like a good thing, except that it means a weak r.f. field must be used so as not to saturate the system.

6. Explain why it is easier to observe NMR in liquids than in solids.

In liquids, T1, the spin-lattice relaxation time is much shorter than solids. One is able to use higher r.f. fields, thereby increasing the signal to noise ratio.

7. What factors of physical processes determine the NMR resonance line width?

The main factor of the resonance line width is due to the spin-spin interaction of the sample. The spin of one proton tends to induce another magnetic filed near another spin, and so on. The resonant frequency for these precessing protons is then shifted, and so is the energy. The result is a broadened band of energy over which the protons may resonate (Melissnos, 354).

8. What is the significance of the g-factor for the proton? If the proton were a Dirac point particle like the electron, what would its g-factor be?

The g-factor for the proton and the electron is a correction factor to make shoddy theory agree with experimental results. For point-like particles QED actually predicts a g-factor of 2, for hadrons like the proton, there is no predicted value.

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;APPENDIX OF DATA AND CALCULATIONS

;Program 1

;gyromagnetic ratios or protons

mag1=[982., 979, 966, 922, 1070, 4240, 4340, 3860, 3370, 4560]
freq1=[4.1716, 4.1636,4.0705, 3.8945, 4.5004, 17.618, 18.029, 16.057, 13.998, 19.012]
mag2=[1663., 1717, 1785, 1905, 1965, 1540, 1424, 1368, 1192]
freq2=[7.0013, 7.2050, 7.50, 8.00, 8.25, 6.50, 6.007, 5.7498, 5.0002]

dmag1=[8., 7, 7, 4, 5, 40, 40, 30, 30, 30]
dfreq1=[0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001]
dfreq2 = dfreq1
dmag2=[5, 4, 4, 6, 5, 4, 4, 7, 10]

fit1=linfit(mag1, freq1)
fit2=linfit(mag2, freq2)

m1=0.00413836
b1=0.0859695
m2=0.00418741
b2=0.0276371

l=findgen(5000)

loadct, 38
set_plot, ’ps’
device, file = ’nmr.ps’, /color ; Or specific file name to which to write

!p.multi = [0,1,2]

plot, mag1, freq1, xtitle=’Magnetic Field (Gauss)              Figure 6.’, $
ytitle=’Frequency (MHz)’, title=’Gyro-Magnetic Ratio for Protons’,$
psym=4

oplot, (l*m1)+b1, linestyle=0

plot, mag2, freq2, xtitle=’Magnetic Field (Gauss)              Figure 7.’, $
ytitle=’Frequency (MHz)’, title=’Gyro-Magnetic Ratio for Mineral Oil (Hydrogen)’,psym=4

oplot, (l*m2)+b2, linestyle=0

device, /close
set_plot, ’x’

!p.multi = [0,1,2]

plot, mag1, freq1, xtitle=’Magnetic Field (Gauss)              Figure 6.’, $
ytitle=’Frequency (MHz)’, title=’Gyro-Magnetic Ratio for Protons’,$
psym=1

oplot, (l*m1)+b1, linestyle=0
for i=0,9 do oplot, [mag1(i),mag1(i)],[freq1(i)-dfreq1(i),freq1(i)+dfreq1(i)]
for i=0,9 do oplot, [mag1(i)-dmag1(i),mag1(i)+dmag1(i)],[freq1(i),freq1(i)]
```

```
plot, mag2, freq2, xtitle='Magnetic Field (Gauss)                Figure 7.’,$
ytitle='Frequency (MHz)’, title='Gyro-Magnetic Ratio for Mineral Oil (Hydrogen)’,psym=1

for i=0,8 do oplot, [mag2(i),mag2(i)],[freq2(i)-dfreq2(i),freq2(i)+dfreq2(i)]
for i=0,8 do oplot, [mag2(i)-dmag2(i),mag2(i)+dmag2(i)],[freq2(i),freq2(i)]

oplot, (l*m2)+b2, linestyle=0

prot=(m1+m2)*2*!pi*10000*1000000/2

;error analysis

magsum1=0
dsqsum1=0
varmag1=0
for i=0,9 do begin
dsqsum1=(freq1(i)*1000000-m1*mag1(i)/10000-b1*1000000)^2 + dsqsum1
magsum1 = mag1(i)/10000 + magsum1
magbar1 = magsum1/10
varmag1 = (mag1(i)/10000-magbar1)^2 + varmag1
dm1sq = dsqsum1/(varmag1*9)
dm1=sqrt(dm1sq)
end

magsum2=0
dsqsum2=0
varmag2=0
for i=0,8 do begin
dsqsum2=(freq2(i)*1000000-m1*mag2(i)/10000-b2*1000000)^2 + dsqsum2
magsum2 = mag2(i)/10000+ magsum2
magbar2 = magsum2/9
varmag2 = (mag2(i)/10000-magbar2)^2 + varmag2
dm2sq = dsqsum2/(varmag2*8)
dm2=sqrt(dm2sq)
end

dmg=(dm1+dm2)/2

end
```

```
;Program 2

;enveloppe of decaying wave

wiggle=fltarr(2,2500)
openr, 6, 'wiggle1.txt'
readf, 6, wiggle
close, 6
time=wiggle(0,*)
volts=wiggle(1,*)

entime=[0.000185, 0.000506, 0.000722, 0.000902, 0.001059]
enamp=[0.3, 0.148, 0.096, 0.084, 0.076]
voltsc=volts-0.0375

enampc=enamp-0.0375
logamp=ALOG(enampc)

new=linfit(entime,logamp)
b=-1.02490
m=-2246.55


loadct, 38
set_plot, 'ps'
device, file = 'env.ps', /color ; Or specific file name to which to write

!p.multi=[0,1,2]

plot, time, voltsc, xtitle='Time (s)          Figure 8.', ytitle='Amplitude (Volts)',$
title='Beat Phenomena'

oplot, [-0.0015,0.0015],[0.0,0.0]
oplot, entime, enampc, psym=4
oplot, time, exp(b)*(exp(m*time))
oplot, time, -1*exp(b)*(exp(m*time))

plot, entime, logamp, xtitle='Time (s)          Figure 9.',$
ytitle='Ln of Amplitude (Volts)', title='Calculation of Initial Amplitudeand Decay Constant', psym=4
oplot, time, m*time+b



device, /close
set_plot, 'x'

!p.multi=[0,1,2]

plot, time, voltsc, xtitle='Time (s)', ytitle='Ln of Amplitude (Volts)',$
title='Beat Phenomena'

oplot, [-0.0015,0.0015],[0.0,0.0]
oplot, entime, enampc, psym=4
oplot, time, exp(b)*(exp(m*time))
oplot, time, -1*exp(b)*(exp(m*time))
```

```
plot, entime, logamp, xtitle='Time (s)',$
ytitle='Ln of Amplitude (Volts)', title='Calculation of Initial Amplitude and Decay Constant', psym=4
oplot, time, m*time+b

;Error analysis

tsum=0
dsqsum=0
vart=0
for i=0,4 do begin
dsqsum =(logamp(i)-m*entime(i)-b)^2 + dsqsum
tsum = entime(i) + tsum
tbar = tsum/5
vart = (entime(i)-tbar)^2 + vart
dmsq = dsqsum/(vart*4)
dm=sqrt(dmsq)
end


end
```