Vibrating Bars and Plates

Theory

Bars and plates, like strings, can vibrate in many different modes, each with its own pattern of nodes and antinodes. Unlike a string, however, the *modal frequencies* of bars and plates are not simple multiples (harmonics) of a fundamental frequency.

In this experiment you will study a few of the many possible modes in a thin metal strip, a rectangular bar, a square plate, a circular plate, and a violin-shaped plate. We will concentrate on the flexural modes of vibration, which can be viewed as patterns of *flexural standing waves* (one-dimensional for strips and bars, two-dimensional for plates). There are also several other types of waves (like longitudinal and torsional) that can propagate in elastic solids.

When a bar or plate is struck, it vibrates in many modes at once. In order to study the individual modes, it is preferable use to a method of selective excitation such as an oscillating magnetic force or a vibrator driven by a function generator.

Procedure

Part I. Aluminum Bar

Hold the aluminum bar about ¼ of the way from the end, and tap it in the center with the rubber mallet. The sound you hear is largely due to the fundamental mode of vibration. Measure this frequency by comparing it to the sound produced by a speaker connected to a function generator. Use a microphone and the sound sensor program on *Data Studio* to do this again, but with the FFT (Fast Fourier Transform). The FFT takes the Fourier transform which gives the acoustic spectrum of your recording.

Now hold the bar at its center and tap about ¼ of the way from one end using the wood of your mallet. This is the first overtone. Measure this frequency. Determine the ratio.

$$\text{ratio} = \frac{f_{1^{\text{st}}\ \text{overtone}}}{f_{\text{fundamental}}}. \qquad (1)$$

Compare it to the theoretical value of 2.76 and determining the percent error.

Place the aluminum bar on the two rubber-band supports so it is supported about ¼ of the way from each end. These are not really fixed supports. With this suspension, the bar will be able to vibrate freely in a variety of modes. Carefully position the mounted coil above and around the small magnet that is glued to the bar. The coil should not be in contact with the magnet since the bar must be free to vibrate. Use the mounted electret microphone to "listen" to the vibrating bar. The microphone should be plugged into the Y-input of the oscilloscope. The microphone cord runs through a tiny preamplifier that needs to be turned on.

Unplug the speaker from the function generator. Connect the coil into the 8 ohm output of the function generator. This coil has a resistance of about 8 ohms but it can overheat very easily. As you search for various resonances, keep the amplitude setting on the function generator *low* until you get near a resonant frequency.

Start with the microphone near the end (opposite the end with the magnet). Watch the oscilloscope as you tune the function generator to the fundamental frequency (i.e. stop when the amplitude of the wave is a maximum). Read the function generator.

Question

1. While the bar is vibrating in its fundamental mode, scan the microphone from one end of the bar to the other. How many nodes are there? You can also sprinkle sand over the bar. Be careful to distinguish between displacement nodes and pressure nodes.

Keep the microphone near the end, measure and tabulate the resonant frequency for the first, second, third and fourth overtones. For each of these, calculate the ratio of the measured frequency divided by the fundamental frequency. Compare your values to the expected values of 2.76, 5.40, 8.93 and 13.34. Why are these the expected values?

Question

2. Go back to the first overtone for a minute and scan the microphone, (or sprinkle sand on the bar) from one end to the other. How many nodes are there? Draw a sketch showing the way the bar is vibrating in this mode.

Turn off the oscilloscope and the function generator. Also, turn off the microphone switch to conserve the battery.

Part II. Metal Strips

Move to the other table where a vibrator is connected to the 8 ohm output of another function generator. Plug the metal strip apparatus into the top of the vibrator.

The frequency of vibration of the fundamental mode for a bar clamped at one end is

$$f_{\text{fundamental}} = \frac{kt}{L^2} \qquad (2)$$

where L is the length of the bar, t is its thickness and k is the stiffness constant.

Starting with the longest one, measure and tabulate the length of all six strips starting from the edge of the nut. Turn on the function generator. Start at about 20 Hz and gradually increase the frequency. Measure and tabulate the fundamental frequency of all six modes. Have the amplitude high enough that you can easily determine the resonant frequencies.

Linearize eq. (2). To do this, define the y and x variables of your graph in such a way that the data forms a line whose slope is kt. Remember to include error bars on your plot. Now that you know the gritty details of the least-squares fit from the Dice experiment, just use origin to fit the line. Calculate stiffness constant k, and its error σ_k.

Find and measure the frequency of the *first overtone* for the longest strip. Theory indicates that it should occur at a frequency about 6.26 times the fundamental. What ratio did you get? Where does 6.26 come from?

Part III. Square Plate

In the early nineteenth century, Ernst Chladni (pronounced Klädne) developed a technique for investigating two dimensional wave patterns. He sprinkled sand onto a plate and used a violin bow to vibrate the plate. The sand will come to rest on nodal lines, where no vibration occurs. Sand that is not resting on a nodal line gets bounced around until it finally lands on a nodal line and comes to rest.

Remove the metal strip apparatus and mount the square plate on the vibrator. Put the vibrator near the center of the poster board sheet and sprinkle a small amount of sand on the plate. Put the plastic cover over the apparatus. Notice that there is a small groove along one of the bottom edges of the box to accommodate the wires.

With the amplitude at a fairly low level, vibrate the plate from about 100 Hz up to 20-30 kHz. As you slowly vary the frequency, you will discover a variety of standing wave patterns. *Don't change the frequency too fast - the resonances are very sharp and you might miss some.* Adjust the amplitude and amount of sand as necessary to get clear patterns. Obtain at least **four** different patterns, photograph the results with a digital camera and indicate the resonant frequency for each pattern.

It may be convenient to resize each of the digital images and insert them into a Word document where it will be easy to label them.

Part IV. Circular Plate

Change to the circular plate with the banana plug attached to the center of the plate. This arrangement will produce symmetrical patterns. Repeat the above process to obtain at least **four** different patterns, photograph the results and indicate the resonant frequency for each pattern. At least one of the four patterns should be at a high frequency to see the contrast.

To obtain asymmetrical patterns, use a Phillips screwdriver to move the banana plug to the off-center hole of the plate. Repeat the above process to obtain at least **two** different patterns, photograph the results and indicate the resonant frequency for each pattern.

Part V. Violin-Shaped Plate

Change to the violin-shaped plate. Obtain at least two different patterns, photograph the results and indicate the resonant frequency for each pattern.

Say something quantitative about the patterns for one of the plates (violin, circular, or square) in your write-up, i.e. are the modes what you should expect for a given frequency? Or do the patterns change from one mode to the next as you may expect? If needed see ref [1] for further detail. You may need to find additional references.

Cleanup

Put the banana plug back into the center hole of the circular plate and put the sand back into the shaker by gently picking up the sheet and using the small funnel.

References

[1] T.D. Rossing, *The Science of Sound.* 3rd Ed., San Francisco: Addison Wesley, 2001.